A FACTORIAL ANALYSIS OF MENTAL TESTS

by

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Thesis presented in fulfilment of the requirements
for the degree of Doctor of Philosophy,
University of Edinburgh.

May 1950.

ACKNOWLEDGMENTS

The Author's thanks are due to Professor Sir Godfrey H. Thomson and Mr. W. G. Emmett, M.A., for advice and encouragement throughout the investigation.

Thanks are also due to:

Mr. R. P. Clark, M.A., B.Ed., Principal Lecturer in Educational Science, Training Centre, Aberdeen.

Mr. J. Maxwell, M.A., B.Ed., Principal Lecturer in Psychology, Moray House.

Mr. J. Sutherland, M.A., B.Ed., Ph.D., Lecturer in Education and Psychology, Moray House.

Mr. Bansi Dhar, M.A., LL.B., D.Litt., D.Phil., Ph.D.

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SECTION I

A FACTORIAL ANALYSIS OF INTELLIGENCE AND PERFORMANCE TESTS USED IN THE POLISH ARMY

I. OBJECT AND SCOPE

The aim of this section is to show the technique of factorial analysis and rotation of factors by the method of extended vectors, and the application of the results of the factorial analysis in selection and vocational guidance.

The secondary aim is to find out the common factors by which the battery of nine tests used in the testing is explained.

The writer was unable to obtain any reliable records about this type of work done in Poland before the war and is unlikely to get this as all documents of previous work were probably destroyed during the war.

This work, however, has been done in the hope that it might prove useful for research workers in Poland in carrying out further investigations. It has been carried out on a sample of 150 Polish soldiers in Britain drawn from 10,000 tested by the personnel of the Polish Psychodemographical Institute* during 1944-46. Although the sample is not representative of the whole Polish population the results which have been obtained may guide investigators in further researches of this type.

* The Polish Psychodemographical Institute was a unit of the Polish Forces consisting of 80 Officers and men and concerned with personnel selection.

II. THE SAMPLE

The Polish Army in Great Britain during 1944-46 amounted to over 60,000 men. This group, however, was not truly representative of the population of Poland as a whole, since 90 per cent of the men came from the western and central parts of the country where conditions are very superior to those in eastern Poland where soil is poor and industries extremely rare. This difference in living conditions has in the course of time had pronounced effects on the inhabitants.

The Polish Psychodemographical Institute was responsible for research in personnel selection and guidance among the Polish Forces in Britain. Six of the tests referred to in this investigation were applied to over 10,000 men, of whom 2 per cent were officers. According to the abilities shown in the tests, the men were posted to suitable units and to duties within their capacity.

During the progress of this work a subsample of 150 subjects was selected and three further tests were given to them in addition to the six already mentioned. Care was taken to make the subsample as representative of the 10,000 as possible, but there was difficulty in making every subject available when required. The tests themselves are described in the next section, but some indication of how far the subsample is representative will be obtained from the following comparative table showing the respective means and standard deviations:-

TABLE I. MEANS AND STANDARD DEVIATIONS

Test	Mean		Standard Deviations	
	Sample	Pop.	Sample	Pop.
1. Raven's Matrix (1938)	23.97	22.81	10.95	10.51
2. Raven's Matrix (1943)	17.55	17.85	7.12	8.01
3. Bennett	25.97	25.30	8.52	8.65
4. Verbal Reasoning	20.02	20.35	9.32	9.55
5. Synonym/Homonym	26.22	25.85	13.56	12.55
6. Rhyme	26.14	25.55	13.31	13.56
7. Cube Constructions	58.62	-	18.36	-
8. Koh's Designs	12.47	-	3.83	-
9. Formboards	30.30	-	13.51	-

III. THE TESTS

The three following groups of tests were used in testing the soldiers of the Polish Army in Great Britain:-

Group I - Non-verbal intelligence tests.

Group II - Verbal tests (in Polish)

Group III - Performance tests.

Each group consists of three tests. The tests of groups I and III were not standardised on the Polish population. Those of group II were standardised on the population of the Polish Army in Great Britain.

Group I consists of:-

1. Raven's Matrix (1938) - A non-verbal test of sound reputation.

2. Raven's (1943) - a non-verbal test.

3. Bennett - a non-verbal test.

Tests 2 and 3 are used in the Army, and, being confidential, no description can be given.

Group II consists of:-

4. Verbal Reasoning Test - This consists of 42 items. In the test the subject is asked to give answers which should be the result of logical thinking.

5. Synonym - Homonym - This consists of 70 items. The subject is asked to write the third word on the dotted line which means nearly the same as the left-hand word, and, in a different sense, means nearly the same as the right-hand word.

e.g. FIRM .FAST. RAPID

6. Rhyme - This consists of 70 items. The subject is asked to write the third word which will rhyme with the second and have the same meaning as the first.

e.g. SADNESS BORROW .SORROW.

Group III /

Group III consists of:-

7. Cube Constructions - Examiner presents model 1 and says: "You see this block. Notice that it is painted on the sides, but not on the top or the bottom; and you see these smaller blocks partly painted and partly unpainted. These nine blocks can be put together so as to make one just like this."

Examiner puts the blocks together, pointing out and commenting on the painted surface of each cube as he fits it in position. He then points to the cube which he has assembled and says: "You see that I have put these cubes together to be just like this big block (pointing to model 1). This one I have made (examiner picks up the nine cubes which he has assembled and holds them together in his hand as one block while he points to the four sides, top and bottom, as he mentions them), is painted on four sides, but not on the top and not on the bottom. You see it is just like this one." (pointing to model 1)

The same procedure is applied with model 2 and model 3.

A record is made of the number of moves, i.e. placements in some position to complete or alter the structure. If parts of a structure are assembled separately, putting such parts together to form the structure as a whole does not count an additional move. If the blocks are fitted together in the hand, the moves are counted just as they are if the blocks are assembled on the table. Changing the face of a block, however, does not count as a move.

Time in seconds for reconstructing each model is also recorded. The time limit for work on each model is two minutes. If the subject assembles the blocks before the time is up, spontaneous corrections are allowed, counting the extra/

extra time and the additional moves. Each block changed counts one move as before. The time should be taken when the subject indicates verbally or otherwise that he has finished.

Whether the subject has finished or not, each misplaced block is counted as three additional moves; each unassembled block six additional moves. No mark is allowed for time if the blocks are not assembled. Time (if the blocks are all assembled), and the total number of moves (whether the blocks are all assembled or not) are credited as follows:-

Models 1, 2	Points	Model 3
Moves		Moves
9	5	8
10-11	4	9-10
12-15	3	11-15
16-25	2	16-25
26-50	1	26-50
Model 1		Models 2, 3
Seconds		Seconds
1-10	5	1-20
11-25	4	21-30
26-50	3	31-50
51-80	2	51-80
81-120	1	81-120

The marks for moves and for time are added for the total score.

8./

8. Koh's Designs - There are sixteen cubes of one inch dimension and all are painted as follows:-

One side red,

One side blue,

One side white,

One side yellow,

One side blue and yellow (divided on the diagonal),

One side red and white (divided diagonally).

The designs are graded in difficulty which increases by modifying the designs at various stages in the following manner:-

(a) By the use of the full colours,

(b) By the use of few diagonaled sides,

(c) By the use of all diagonaled sides,

(d) By turning the design on one of its corners,

(e) By eliminating the outside boundary line,

(f) By increasing the number of blocks to be used.

(g) By increasing dissymmetery in design,

(h) By decreasing the number of different colours used in each design.

The designs from No. 1 to No. 9 are inch square,

No. 10 to No. 11 are one and a half inch square,

No. 12 to No.17 are two inches square.

To clarify the method of scoring one illustration will be utilized. For example, Design No. 2 has a score value of 5. This full amount is attained if a reagent completes the design successfully in less than thirty-one seconds and with less than seven moves. If thirty-one or more seconds are utilized one/

one point is deducted from the score, and if seven or more moves are made an additional point is deducted.

KOH'S SCORING CARD

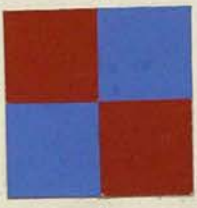
Design No.	Score Value Points	Points to be subtracted Excess time		Excess Moves 1 point
		1 point	2 points	
1	3	21 secs and over		6 and over
2	5	31 " " "		7 " "
3	6	21 " to 35	36 secs and over	8 " "
4	7	31 " to 1 min.00	1 min. 1 sec.	10 " "
5	7	36 " to 1 " 5	1 " 6 "	11 " "
6	7	36 " to 1 " 00	1 " 1 "	12 " "
7	7	41 " to 1 " 10	1 " 11 "	11 " "
8	8	41 " to - " 55	- " 56 "	10 " "
9	9	56 " to 1 " 10	1 " 11 "	15 " "
10	9	1 min 56 " to 2 " 10	2 " 11 "	22 " "
11	8	1 min 46 " to 2 " 30	2 " 31 "	19 " "
12	9	2 " 26 " to 2 " 40	2 " 41 "	30 " "
13	9	2 " 21 " to 2 " 33	2 " 34 "	31 " "
14	9	2 " 26 " to 2 " 40	2 " 41 "	32 " "
15	9	2 " 41 " to 3 " 00	3 " 1 "	32 " "
16	10	2 " 41 " to 3 " 05	3 " 6 "	31 " "
17	11	2 " 41 " to 2 " 55	2 " 56 "	30 " "

Time limits for each design

Design No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Time limit in minutes	1½	1½	1½	2	2	2	2	2	2	3	3½	3½	3½	3½	4	4	4

KOH'S DESIGNS

TRIAL



1

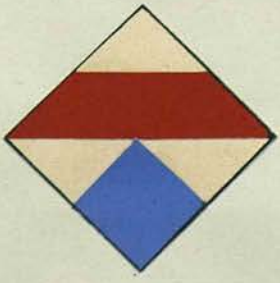
2

3

4

5

6



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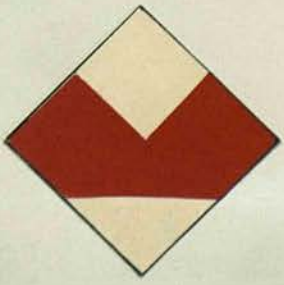

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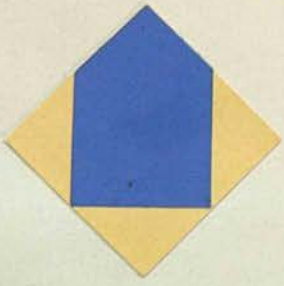

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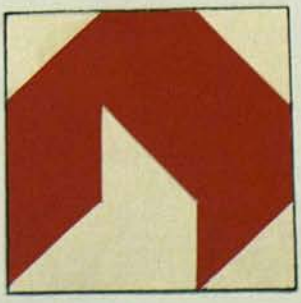

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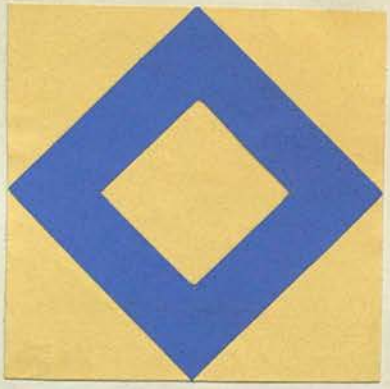
KOH'S DESIGNS.(continued)

12

13



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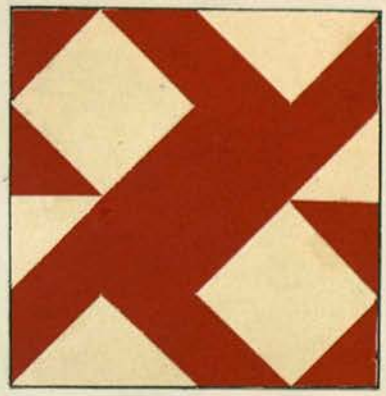

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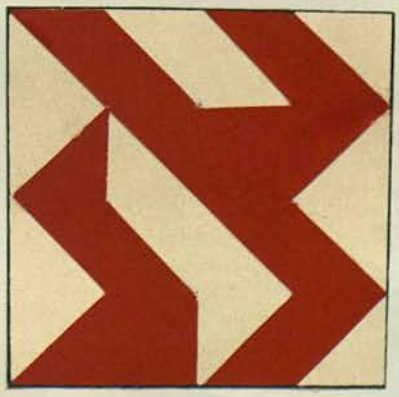

16

17

9. Formboards (Kent-Shakow) - This consists of a wooden board out of which pieces of various geometric shapes have been cut. These pieces some of which have been cut into several smaller pieces have to be replaced by the testee. The formboard is placed before the subject with the pieces arranged in front, and at a given signal he replaces the pieces as quickly as possible. The time taken by him to do this is recorded by stopwatch. The following table gives the scoring system:-

	1 2S	2 2D	3 3S	4 3D	5 4S	6 4D
to 15"	6	8				
16-30"	5	7				
31-45"	4	6	10	16	12	
46-60"	3	5	9	15	11	
61-90"	2	4	8	14	10	20
91-120"	1	3	7	13	9	19
121-150"			6	12	8	18
151-180"			5	11	7	17
181-240"				10	6	16
241-300"				9		15
301-360"						14
361-420"						13

IV. THE ANALYSIS

Rank of a Matrix

Professor L. L. Thurstone generalised Professor C. Spearman's Two-factor Theory to n dimensions. Under the multiple factor hypothesis a test score can be expressed as a linear function of a number of factors. The inter-correlations of a group of tests can be explained in terms of a number of correlated or orthogonal factors and the variance of each test can be accounted for by these factors and a specific factor. The number of dependent or independent factors represented by the inter-correlations of 'n' tests is equal to the rank of their correlational matrix. A table of inter-correlations of tests may thus be thought of as a matrix. The rank of a matrix is defined to be the order of the highest non-vanishing determinant from the matrix. If all the tetrad differences, after one factor has been extracted, vanish, the rank of the correlation matrix is ONE, and each of these tests is explained by one factor and a specific. In multiple factor analysis the number of common factors represented by the inter-correlations of a battery of tests would be equal to the order of the highest non-vanishing determinant in the correlational matrix. Spearman and Holzinger (ref. 5) established a formula known as 16A:

$$\frac{2}{(N)^{\frac{1}{2}}} \left[r^2(1-r)^2 + (1-R)s^2 \right]^{\frac{1}{2}}$$

where N = number of persons in the sample,

r = the mean of all the r's in the whole table,

s^2 = their mean squared deviation from r,

$R = 3r\frac{n-4}{n-2} - 2r\frac{n-6}{n-2}$, and

n = number of tests.

If/

If the standard deviation of the tetrad differences is smaller than or equal to the probable error obtained from 16A formula, the set of inter-correlations is explained entirely by one factor only and specific factor. The following formula gives the number of tetrad differences:

$$n(n - 1)(n - 2)(n - 3)/8$$

Geometric Interpretation of Correlation

The tests may be considered as vectors with a common origin. A vector may be defined as a line with a given magnitude and direction. The magnitude of a test vector is given by the square root of the communality of a test. The correlation between two tests is given by the scalar product of their vectors, i.e. the product of the length of the vectors by the cosine of their angular separation. The cosines of the angles between the test vector and the factors are known as direction cosines. For simplicity an example is carried out of two tests which are described in terms of two common uncorrelated factors. Let us assume a part of the factor pattern as follows:-

$$t_1 = a_1 I + b_1 II$$

$$t_2 = a_2 I + b_2 II$$

where I and II are the factors and a_1, a_2, b_1, b_2 are the coefficients. Then the correlation between these two tests is given by

$$r_{12} = a_1 a_2 + b_1 b_2$$

In/

In the following diagram the two test vectors are represented by t_1 and t_2. Their coordinates on the two orthogonal axes repreeenting I and II are a_1b_1 and a_2b_2 respectively.

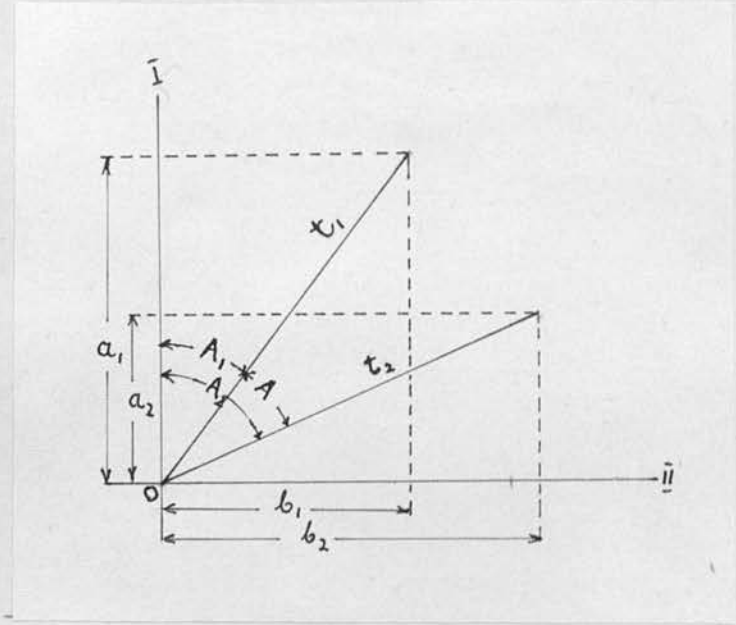

Diagram I -
Geometric Interpretation of Correlation

h_1 = length of the vector t_1

and h_2 = length of the vector t_2

then $h_1 = (a_1^2 + b_1^2)^{\frac{1}{2}}$

and $h_2 = (a_2^2 + b_2^2)^{\frac{1}{2}}$

A_1 = angle between test vector t_1 and factor I, and

A_2 = angle between test vector t_2 and factor I

A = the angle between the test vector t_1 and test vector t_2.

Now, applying the Cosine Law, we get

$$\cos A = \cos A_1 \cos A_2 + \sin A_1 \sin A_2$$

But $\cos A_1 = \frac{a_1}{h_1}$ and $\sin A_1 = \frac{b_1}{h_1}$

$\cos A_2 = \frac{a_2}{h_2}$ and $\sin A_2 = \frac{b_2}{h_2}$

Substituting/

Substituting these values in the above equation we get

$$\cos A = \frac{a_1 a_2 + b_1 b_2}{h_1 h_2}$$

finally, $r_{12} = h_1 h_2 \cos A$

If we consider communality as 1, the above formula becomes

$$r_{12} = \cos A$$

If the angle between test vectors is 90°, the cos of 90° equals zero, and hence the correlation between these two variables is zero, or, in other words, they are orthogonal or uncorrelated. Generalisation of this case to n dimensions will be evident.

Centroid Solution

The first task in multiple factor analysis is to obtain any factor pattern with uncorrelated factors. There are several ways of obtaining this initial factor pattern. These are:-

Centroid Analysis or Simple Summation Method

Hotelling's Principal Components

Lawley's Maximum Likelihood Method

Holzinger's Bi-factor Analysis.

In this work the Centroid Method was applied. In a Centroid Solution the axes are orthogonal. The full description of the calculation of a Centroid Solution is given in the next section. Here a short procedure for obtaining a Centroid pattern is given.

The communal parts of the tests are represented by a set of n vectors with a common origin in a space of k dimensions, where k is the number of common factors, and the scalar product/

product of any pair of these vectors is the correlation between the tests. The communal parts of the tests may be also represented by the k coordinates of the end points of these vectors on orthogonal axes. The set of axes can be rotated so that the centroid of the system of n points lies in the first axis of reference. Thus it is possible to obtain the projection of each of the test vectors, or the coordinate of each test in the first axis of reference through the centroid.

The next step is to get the first factor residuals from which the second coordinates are found. The residual correlations may be regarded as the scalar products of pairs of residual vectors in a space of k-1 dimensions. Since the k-1 coordinates of the centroid of the residual vectors are zero, the centroid is at the origin in the k-1 space. In order to get the next factor residuals the same process is applied.

TABLE II

MATRIX OF INTER-CORRELATIONS OF NINE TESTS WITH GUESSED COMMUNALITIES - R

	1	2	3	4	5	6	7	8	9
1.	(6316)	6020	6316	0197	1966	-0729	3279	4468	5207
2.	6020	(6833)	6833	1508	3554	2347	4544	0360	1622
3.	6316	6833	(6833)	2774	1942	-0835	3811	1177	3799
4.	0197	1508	2774	(5616)	5616	3932	4418	0648	0809
5.	1966	3554	1942	5616	(5830)	5830	3707	-0264	1002
6.	-0729	2347	-0835	3932	5830	(5830)	3573	-0562	-1030
7.	3279	4544	3811	4418	3707	3573	(5340)	5340	4462
8.	4468	0360	1177	0648	-0264	-0562	5340	(5340)	4697
9.	5207	1622	3799	0809	1002	-1030	4462	4697	(5207)

NOTE. The decimal points should be understood.

From this matrix R by Thurstone's Centroid Method three centroid factors were obtained (for calculation see App. 2).

TABLE III CENTROID LOADINGS - F_c

Test	I	II	III	h^2
1. Raven's Matrix (1938)	.6507	.5869	.1798	.8002
2. Raven's Matrix (1943)	.6622	.1545	.5101	.7226
3. Bennett	.6430	.3879	.2308	.6172
4. Verbal Reasoning	.5026	-.4970	-.0685	.5043
5. Synonym/Homonym	.5747	-.5223	.2617	.6716
6. Rhyme	.3615	-.6310	.1734	.5589
7. Cube Constructions	.7577	-.0980	-.3382	.6981
8. Koh's Designs	.4176	.2064	-.5106	.4777
9. Formboards	.5076	.3508	-.3470	.5011
Variance taken out	.3328	.1790	.1049	.6167

Factor I has all positive loadings; factor II has negative loadings in tests 4, 5, 6 and 7; factor III has negative loadings in tests 4, 7, 8 and 9. There is a difficulty in the interpretation of negative loadings in the matrix F_c. Thurstone says, "One can understand how the possession of a certain ability can aid in a test performance and can imagine that an ability has no effect on a test performance, but it is difficult to think of an ability that is detrimental in many test performances." Therefore the centroid matrix F_c must be rotated to get a configuration called by Thurstone "Simple Structure." The definition of simple structure given by Thurstone (ref. 7) is "a structure in which each trait vector is contained in one or more of the orthogonal (or/

(or oblique) co-ordinate hyperplanes will be called an orthogonal (or oblique) simple structure." That is to say each column must have at least as many zeros (or near zero) as there are columns; all loadings must be positive and each row must have at least one zero. The following table gives an example of a simple structure:-

TABLE IV. FACTOR PATTERN

	I	II	III
1.		x	x
2.	x	x	
3.		x	x
4.	x		x
5.	x	x	
6.	x		
7.	x		x
8.			x
9.		x	x

In order to get simple structure a new rotational method, devised by Thurstone, was used, which takes three factors at a time (ref. 7). For the rotation of a centroid matrix the following computation must be made. Extension of the loadings of the centroid factors, i.e. to make the projections of all tests on centroid factor I equal to unity. It is got by multiplying each row of the matrix F_c by the reciprocal of its first member. The matrix of extended loadings is then:-

TABLE V. EXTENDED LOADINGS - F_e

	I_e	II_e	III_e
1.	1.0000	.9019	.2763
2.	1.0000	.2333	.7703
3.	1.0000	.6033	.3589
4.	1.0000	-.9889	-.1363
5.	1.0000	-.9088	.4554
6.	1.0000	-1.7455	.4797
7.	1.0000	-.1293	-.4463
8.	1.0000	.4942	-1.2227
9.	1.0000	.6911	-.6836

The following diagram shows the extensions of the test vectors. The lengths of the test vectors are determined by the square roots of the communalities. By this extension a pattern of dots was obtained on a plane II_eIII_e, which is at unit distance along axis I_e and at right angles to it.

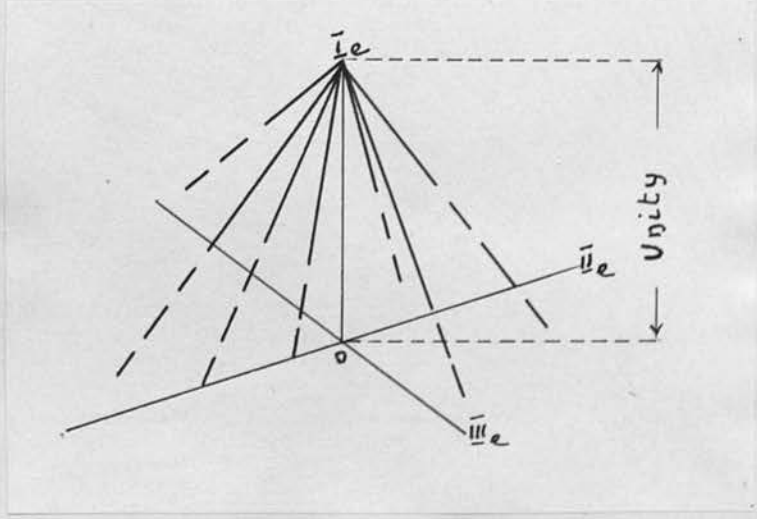

Diagram II -
Geometric Interpretation of Extended Vectors

The/

The columns II_e and III_e of the matrix F_e are the co-ordinates of the dots on the above-mentioned plane. When a diagram of these dots is made the following figure is obtained:-

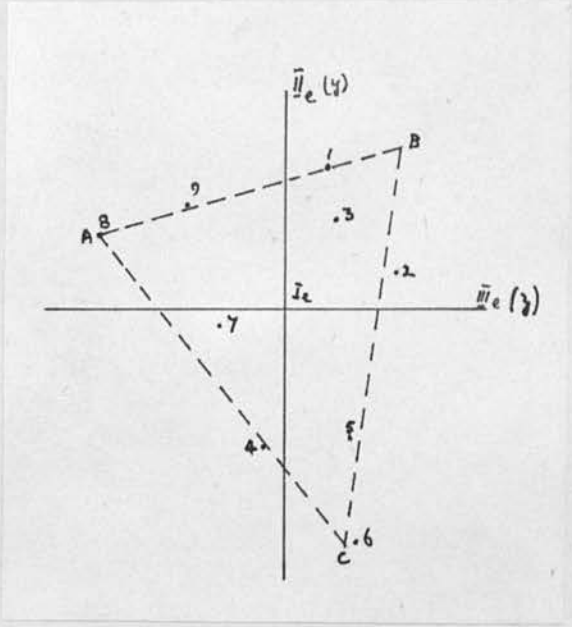

Diagram III -
Configuration of Test Points

Having plotted the points shown in the above diagram our next task is to draw the lines shown. These are drawn in such a way, that they pass through or near to as many points as possible producing a triangle. In drawing the lines care should be taken not to leave any points far inside the triangle as it would mean that these tests would have negative loadings in some of the factors. A point falling outside the triangle represents a test which involves all three factors and has positive loadings in each. From the above diagram the equations of the lines (or rather of the planes through each line and through the origin) were found as follows:-

(AB)/

(AB) $1.2393x - 1.4990y + .4077z = 0$

(AC) $1.8526x + 1.6559y + 2.1845z = 0$

(BC) $1.9632x + .4399y - 2.8153z = 0$

The coefficients of each equation have then to be normalised, i.e. reduced proportionately so that their sum of squares is unity, and they are then direction cosines of lines (called reference vectors) at right angles in each case to a plane passing through the line (e.g. the line AB) and the origin. These coefficients, after being normalised, are then written as columns in the following matrix:-

M

.6236	.5599	.5673
-.7543	.5005	.1271
.2052	.6603	-.8136

This matrix gives the cosines of the angles between reference vectors and centroid axes. By the reference vectors Thurstone means the imaginary tests which are uncorrelated each with a group of the mental tests, which are on the plane to which the reference vector is at right angle.. When the factors are oblique the reference vectors differ from the primary fectors. The primary factors are defined by the lines of the inter-sections of the planes (AOB, AOC, BOC). In our case, since there are only three primary factors, they are simply three of the edges of a tetrahedron. It is shown in the following diagram:-

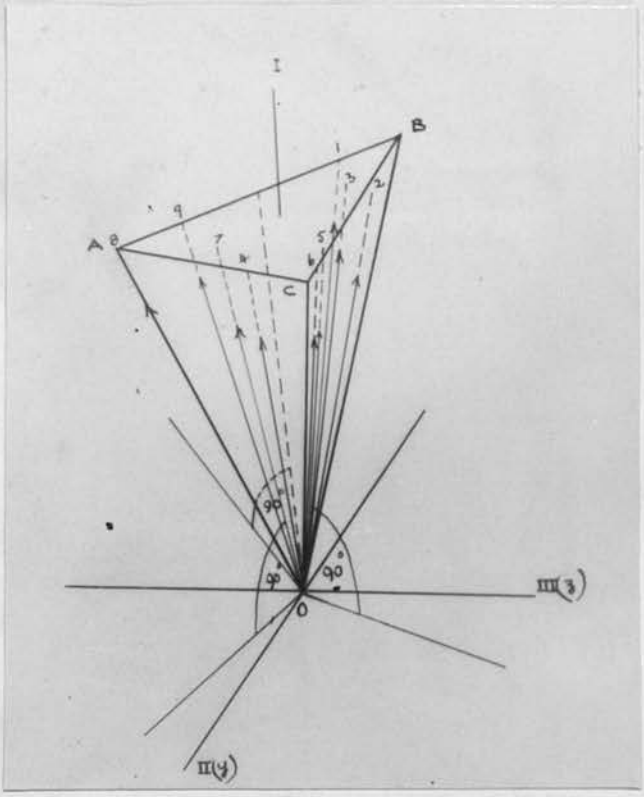

Diagram IV. -
Geometric Interpretation of Primary Factors.

The matrix of the centroid loadings F_c must now be post-multiplied by the rotating matrix M to obtain the projections of the test vectors on the reference vectors (simple structure on the reference vectors). That is to say

$$F_c \times M = V$$

TABLE VI./

TABLE VI

SIMPLE STRUCTURE ON REFERENCE VECTORS - V

	I	II	III
1.	,0000	.7768	.2975
2.	.4011	.7849	-.0197
3.	.1556	.7066	.2263
4.	.6743	-.0126	.2777
5.	.8061	.2332	.0467
6.	.7370	.0011	-.0162
7.	.4770	.1519	.6925
8.	0000	0000	.6787
9.	-.0193	.2306	.6149

The above matrix is a simple structure on the reference vectors. It has a zero (or near zero) in each row, and as many zeros or more in each column as there are columns.

It seems psychologically reasonable that each of these tests will not require all of the abilities that are required by the test battery as a whole. If there is a distinct ability to deal with words, then that factor would not be expected in the picture or performance tests. After the factorial matrix F_c has been rotated all negative loadings disappear and by the same time many entries became zero or very near. Such a matrix can be interpreted psychologically. The entries in a simple structure are the projections of the test vectors on the reference vectors, but the reference vectors are not the same as primary factors, therefore the entries in a structure on primary factors would be different.

The/

The difference between reference vectors and primary factors can be easily seen when the angles between reference vectors and the angles between primary factors are found.

The cosines of the angles between reference vectors are got by pre-multiplying the rotating matrix M by its transpose.

$$M' \times M = K$$

$$\underline{K}$$

1.0000	.1071	.0909
.1071	1.0000	-.1560
.0909	-.1560	1.0000

Matrix K gives the cosines of the angles between the reference vectors, and the angles are:-

	I	II	III
I		$83^{o}50'$	$84^{o}46'$
II			$98^{o}58'$
III			

To find the angles between the primary factors is to pre- and post-multiply the reciprocal of the matrix K by a diagonal matrix D. The elements of thediagonal matrix D are the reciprocals of the square roots of the diagonal elements of the reciprocal matrix K (for the calculation of reciprocal of a matrix see Appendix 3).

$$\underline{K^{-1}}$$

1.0239	-.1272	-.1128
-.1272	1.0407	.1738
.-.1128	.1738	1.0373

$$\underline{D}\ /$$

$\underline{D}$

$(1.0239)^{-\frac{1}{2}}$

$(1.0407)^{-\frac{1}{2}}$

$(1.0373)^{-\frac{1}{2}}$

When solved

$\underline{D}$

.9883

.9802

.9819

$\underline{DK^{-1}D}$

1.0000	-.1233	-.1096
-.1233	1.0000	.1674
-.1096	.1674	1.0000

The matrix $DK^{-1}D$ gives the cosines of the angles between the primary factors, and the angles are:-

	I	II	III
I		$97^{o}05'$	$96^{o}17'$
II			$80^{o}20'$
III			

The projections of the test vectors on the primary factors will differ from the projections on the reference vectors. The structure on the primary factors will not be a simple configuration, and it is found by post-multiplying the factorial matrix F_c by the reciprocal of the transposed matrix M and by the above-mentioned diagonal matrix D.

TABLE VII/

TABLE VII

STRUCTURE ON PRIMARY FACTORS - $F_c(M')^{-1}D$

	I	II	III
1.	-.1311	.8437	.4360
2.	.3095	.7478	.0696
3.	.0434	.7404	.3342
4.	.6532	-.0497	.2060
5.	.7815	.1453	-.0020
6.	.7478	-.0938	-.0982
7.	.3862	.2135	.6789
8.	-.0758	.1157	.6916
9.	-.1172	.3427	.6683

The entries of this matrix are the correlation coefficients between the primary factors and the tests, and they are used in finding the prediction equations.

Now to distinguish between pattern and structure Professor G. H. Thomson (ref. 6) says: "A structure is a table of correlation coefficients, a pattern is a table of coefficients in a specification equation specifying how a test score is made up by factors. The entries in a pattern are loadings or saturations of the tests with the factors, but not correlation coefficients." The pattern on primary factors is shown below:-

TABLE VIII/

TABLE VIII

PATTERN ON PRIMARY FACTORS - F_cMD^{-1}

	I	II	III
1.	0000	.7924	.3030
2.	.4059	.8007	-.0201
3.	.1576	.7208	.2305
4.	.6823	-.0129	.2828
5.	.8157	.2379	.0476
6.	.7458	.0011	-.0165
7.	.4827	.1550	.7053
8.	0000	0000	.6915
9.	-.0195	.2352	.6263

In the case of orthogonality of the primary factors there is no difference between pattern and structure. When the pattern on primary factors is post-multiplied by the transpose of structure on primary factors the original matrix of correlation is obtained, if the last factor residuals are zero. The same procedure can be applied to reference vectors.

$$FMD^{-1} \times \left[F(M')^{-1}D\right]' = FF' = R$$

$$F(M')^{-1} \times (FM)' = FF' = R$$

This can be used as a check on calculations. The geometric representation of it is shown in the following diagram:-

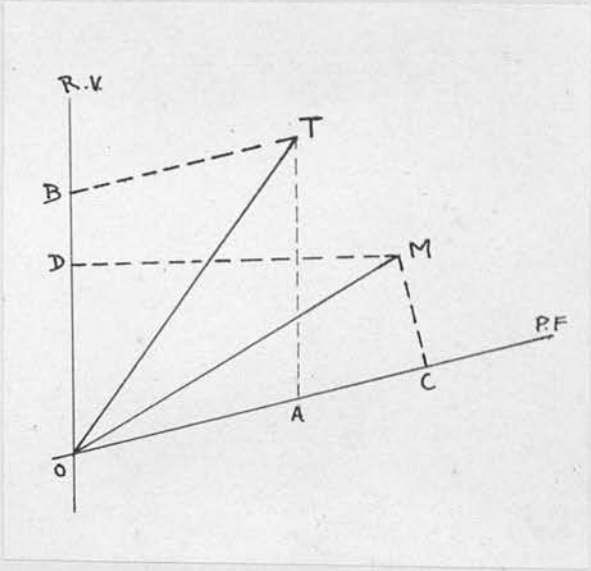

Diagram V -
Geometric Interpretation of
Equation: $F_c MD^{-1} \times \left[F_c (M')^{-1} D\right]' = R$

The diagram represents a space defined by two oblique axes, with origin O, and two points T and M each at unit distance from the origin O. The lines OT and OM may be taken as test vectors and cos MOT - correlation between them. The axes represent Reference Vector (RV-O) and its Primary Factor (PF-O). The oblique coordinates OB and OA of point T are the entries of the pattern on reference vector and primary factor respectively. OD and OC are the entries of the structure on reference vector and primary factor respectively, and they are the projection of OM on the axes. OM being unity, they are the cosines. Then the inner product of those oblique co-ordinates of T with these cosines adds up to the projection of OT on OM, i.e. to cos MOT or the correlation coefficient.

V. SECOND ORDER FACTORS

Professor L. L.Thurstone calls the factors which were obtained from the original matrix of inter-correlations first order factors, whether they are correlated or not. If there is correlation between first order factors he makes a further analysis.*

In this work the primary factors are correlated and the correlations between them are given by the following matrix:-

$$D(M'M)^{-1}D$$

1	-.1233	-.1096
-.1233	1	.1674
-.1096	.1674	1

From this matrix of correlation, treating the primary factors as three tests, we get by Spearman's method a general factor g' and specific s',

$$g_a = \left[\frac{r_{ab}r_{ac}}{r_{bc}}\right]^{\frac{1}{2}}$$

$$B$$

.2840	.9588	
.4399		.9009
.3857		.9226

Matrix B gives the loadings of second order g' and s'. The general factor appears in each test and specific different in each.

* If some of the correlations between first order factors are very small, the Heywood Case may take place. This is the case where we obtain a saturation with g greater than unity, which is impossible.

The factorial matrix F_c has as many rows as tests and as many columns as centroid factors. Each row gives the direction cosines of a test vector with the centroid axes. Thurstone extends F_c in length downwards by adding more rows for the primary factors, treating each primary factor as if it were a test. This extension of F_c is square and the whole matrix is now

$$\begin{matrix} F_c \\ \vdots \\ T \end{matrix}$$

The matrix M is such a matrix that F_cM has the properties of simple structure. It rotates the centroid axes to the new positions which in this work are oblique. These oblique axes are called the reference vectors and the columns of matrix M are their direction cosines with the centroid axes (which are orthogonal). The direction cosines of the primary factors are given by the T-part which can be found by pre-multiplying the reciprocal of a matrix M by the diagonal matrix D, i.e.

$$T = DM^{-1}$$

When the T-part of $\begin{matrix} F_c \\ \vdots \\ T \end{matrix}$ is post-multiplied by M the result is the diagonal matrix D.

$$\begin{matrix} F_c \\ \vdots \\ T \end{matrix} \times M = \frac{V}{D}$$

In order to get the loadings with the second order factors, the matrix $\begin{matrix} F_c \\ \vdots \\ T \end{matrix}$ must be post-multiplied by an unknown matrix K. But it is known that T post-multiplied by the unknown matrix K gives matrix B, i.e.

$$T \times K = B$$

$$\text{So} \quad K = T^{-1} \times B$$

$$\text{But} \quad T = DM^{-1}$$

$$\text{So} \quad K = (DM^{-1})^{-1} B = MD^{-1} B$$

The/

The required loadings of the original tests in the second order factors are therefore

$$F_c K = F_c M D^{-1} B$$

But $F_c M = V$ (simple structure)

So $F_c K = V D^{-1} B = G$ (matrix of loadings of the second order factors)

The following matrix G consists of four second order factors, namely, factor g common to all tests, and three factors which are similar to the factors in the simple structure, i.e. g plus orthogonal simple structure of three factors.

TABLE IX. SECOND ORDER FACTORS

G

	g'	I'	II'	III'
1.	.4602	0000	.7139	.2796
2.	.4539	.3878	.7214	-.0185
3.	.4482	.1505	.6494	.2127
4.	.3046	.6519	-.0116	.2610
5.	.3622	.7794	.2143	.0439
6.	.2141	.7126	.0001	-.0152
7.	.3815	.3613	.1396	.6509
8.	.2664	0000	0000	.6378
9.	.3377	-.0186	.2119	.5779

VI. INTERPRETATION OF FACTORS

A factor, says Burt, (ref. 2) is simply an average or sum total of certain measurements empirically obtained. And the process of obtaining factors is to enable us to get a tentative basis of classification of variables. In Grammar we make a distinction between a class name like "Man" and the abstract noun "Manliness." This is exactly what we do in the process of factorisation. The tests or individuals are considered in terms of the qualities or traits that are material to the investigation; and it is the abstract and not the concrete aspect that is important for purposes of synthesis. This abstraction is reached by examining the correlation between the forms of behaviour. How far the factors represent a real basis of classification corresponding to ability is a matter on which opinions are divided. According to Thomson they are mathematical entities and he warns us against the danger of reifying them. This would be true of factorisation in most cases. By a careful selection of tests and their sifting after factorial analysis, we obtain factors which enable us to classify human abilities as found in a certain population with specific reference to some human activity. It is an arduous process but if we attempt it we have to be scientific in our psychological diagnosis. Thurstone's use of Primary Factors is a significant departure and an attempt in the right direction to minimise vagueness. The process of factorisation, according to him, raises two problems:-

(1) isolating a primary factor and determining by repeated experiments that it has some functional uniqueness,

(2)/

(2) finding the psychological or physiological meaning of a functional uniqueness when it has been determined.

To isolate a primary factor we require not only knowledge of mathematical technique but also insight into human activity in the sphere of our investigations. And it is this insight which helps us in naming these factors when isolated. Mere mathematical technique is no substitute for the necessary insight.

In this work it has been shown that the set of inter-correlations of nine tests can be analysed into three group factors each accounting for the following variances:-

I	.3328
II	.1790
III	.1049

These factors were rotated to an oblique structure and then interpreted, as follows:-

The first factor in the battery of the tests which appears in tests 4, 5 and 6 is clearly a verbal factor. It appears also in tests 2 and 7 but has comparatively low loadings.

The second factor appears in tests 1, 2 and 3. These tests are independent of language and training but are yet in no sense performance tests. They measure eductive ability which is definable as the essential creative factor in intelligent conduct.

The third factor was found to have high loadings in tests 7, 8 and 9. This very definitely suggests that it is a factor involving ability to perform a process of trial and error/

error by mental imagery rather than by a recourse to actual handling of material. W. P. Alexander (ref. 1) calls this factor, which always appears in the performance tests, practical ability.

VII. SELECTION AND GUIDANCE USING REGRESSION EQUATIONS TO ESTIMATE FACTORS

The desirability of being able to grade the army recruits with a high degree of accuracy was so apparent that the first attempts to measure the specialised abilities requisite for special occupations were for the purpose of vocational selection. If every recruit could be selected for that occupation in the Services for which he is best fitted the subsequent task of training would be facilitated, better results would accrue, and a considerable saving of time would be effected in producing skilled men. It has been realised, however, that the problem is just as important from the standpoint of vocational guidance when the trained soldier is about to return to civil life. Vocational guidance and vocational selection both seek to secure a harmonious adjustment to occupational life, and both recognise that it must be achieved early rather than late, if possible at the very commencement of the career. The recruiting offices have difficulty in finding a reliable criterion as they usually have very little or no information about the recruits. Here factorial analysis of the tests which were given to these recruits comes in to help. The results of factorial analysis could be applied in both vocational guidance and vocational selection. Professor G. H. Thomson in the article "The advantages and disadvantages of using factors in vocational guidance" (ref. 6) gives the following analogy:-

> "To change to a simpler analogy, the regression equation method is like an American buying American goods from another American in dollars. The factor method is as though the buyer first unnecessarily changed/

changed his dollars into francs, and then asked the seller to quote him a price in francs. He is sure to lose on the roundabout method, probably lose money and certainly lose time. The factor method will, however, be useful when a medium of exchange is essential, as when in our analogy the buyer has one coinage and the seller, a foreigner, has another. That is when one set of tests has been used for analysing the occupation and a different set has been given to the candidate. This is not an ideal situation, but it is one which will frequently arise, for example, when we wish to practise not merely vocational selection but vocational guidance. In selecting men for a given occupation we are almost certain to use the same tests on the candidates as on the occupation. But if we wish to advise the rejected men about other occupations, we are less likely to be in this mathematically most favourable position, and we may have to use factors as go-betweens."

In this work the factor method was used. The battery of nine tests, which was given to 150 subjects, was analysed. As a result of the analysis three factors were obtained. As a criterion the structure on the primary factors $F_c(M')^{-1}D$, which consists of correlation coefficients between the primary factors and the tests, was used.

In order to measure these three factors in any person the best loadings of the tests for each factor must be found. The principal part of the above is to find the reciprocal of a matrix (R) of the original correlation coefficients.

TABLE X /

TABLE X

RECIPROCAL OF ORIGINAL CORRELATION MATRIX - R^{-1}

	1	2	3	4	5	6	7	8	9
1	3.7580	-1.8446	-1.0360	.3580	-.6475	.1264	1.4676	-1.8335	-1.0087
2	-1.8446	3.8991	-1.6057	.9025	-.3203	-.5446	-1.7632	1.2218	1.0535
3	-1.0360	-1.6057	3.2515	-1.0961	.4404	.8298	-.2705	.6388	-.4843
4	.3580	.9025	-1.0961	2.1031	-.9797	-.2394	-.7320	.0064	.3102
5	-.6475	-.3203	.4404	-.9797	2.2450	-.8277	-.0702	.4517	-.1897
6	.1264	-.5446	.8298	-.2394	-.8277	2.0415	-.6322	.2171	.2001
7	1.4676	-1.7632	-.2705	-.7320	-.0702	-.6322	3.4873	-1.9319	-1.0227
8	-1.8335	1.2218	.6388	.0064	.4517	.2171	-1.9319	2.7196	.0750
9	-1.0087	1.0535	-.4843	.3102	-.1897	.2001	-1.0227	.0750	1.9738

Having found R^{-1} the next step is to find the regression coefficients for each factor with each test. This is done by:-

$$d'R^{-1}$$

for each factor respectively, where d' is a row of entries which are in the columns of matrix $F_c(M')^{-1}D$, and R^{-1} is a reciprocal of the original matrix of correlations.

The regression coefficients found, as shown above, are:-

TABLE XI. REGRESSION COEFFICIENTS

	I	II	III
1	-.4599	.6557	.2807
2	.4122	.2557	-.4775
3	-.0672	.2234	.1495
4	.2941	-.1084	.0260
5	.4609	-.0351	-.1444
6	.2899	-.0275	-.0986
7	-.1409	-.0243	.6857
8	.2034	-.1873	.0978
9	.0084	-.0168	.1927

When/

When checked by pooling square:-

For factor I

1	.883
.883	.883

For factor II

1	.880
.880	.880

For factor III

1	.816
.816	.816

The multiple correlation coefficients between estimated and true value are obtained by:-

$$r_m^2 = d'R^{-1}d$$

and they are:-

I $r_m = .9394$

II $r_m = .9381$

III $r_m = .9033$

To test the significance of the regression coefficients their standard errors must be found . To find the standard errors of the regression coefficients one must first multiply matrix R^{-1} by the following ratio:-

$$(1 - d'R^{-1}d) / (N - p - 1)$$

where N = number of people tested

P = number of tests

N - p - 1 = degree of freedom.

In this case the ratios are:-

For factor I .00084

II .00086

III .00131

The/

The products consist of the variances (along the diagonal cells) and covariances of the regression coefficients for each factor respectively. The square roots of the variances are then the required standard errors, and they are:-

TABLE XII. STANDARD ERRORS OF REGRESSION COEFFICIENTS

For factor I:-

Tests	Regr. Coeff.	St. Error	Sign. at 1% level
1	-.4599	.0566	yes
2	.4122	.0574	yes
3	-.0672	.0520	no
4	.2941	.0424	yes
5	.4609	.0436	yes
6	.2899	.0412	yes
7	-.1419	.0539	no
8	.2034	.0480	yes
9	.0084	.0412	no

For factor II:-

Tests	Regr. Coeff.	St. Error	Sign. at 1% level
1	.6557	.0566	yes
2	.2557	.0583	yes
3	.2234	.0529	yes
4	-.1084	.0424	no
5	-.0351	.0436	no
6	-.0275	.0424	no
7	-.0243	.0548	no
8	-.1873	.0480	yes
9	-.0168	.0412	no

For/

For factor III:-

Tests	Regr. Coeff.	St. Error	Sign. at 1% level
1	.2807	.0700	yes
2	-.4775	.0714	yes
3	.1495	.0648	no
4	.0260	.0520	no
5	-.1444	.0529	yes
6	-.0986	.0520	no
7	.6857	.0671	yes
8	.0978	.0592	no
9	.1927	.0510	yes

For testing the significance Fisher and Yates statistical tables were used. (Table III distribution of t page 26).

It is possible now to estimate the amount of each factor possessed by any man. For that the following prediction equation is used:-

$$z_o = b_1 z_1 + b_2 z_2 + \ldots\ldots + b_{n-1} z_{n-1} + b_n z_n$$

In this work the subjects number 1, 71, 92, 100, and 150 were chosen as typical, and their best predicted scores in each factor were found. (For number of subjects see Appendix 1, and for the calculation of best predicted scores see Appendix 4)

Their profiles are illustrated in the following diagrams:-

Subject No. 1
(Engineers - all kinds)

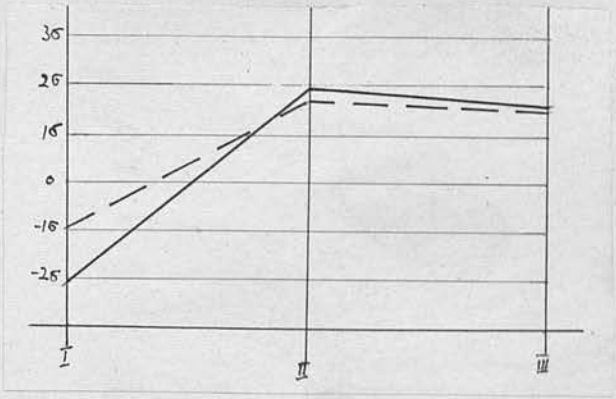

Diagram VI

Subject No. 71
(Signals)

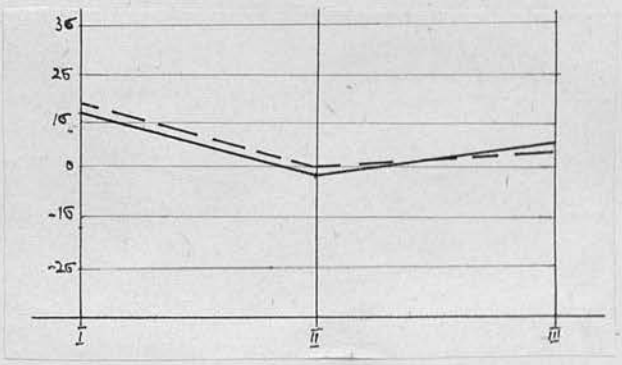

Diagram VII

Subject No. 92
(Demolition)

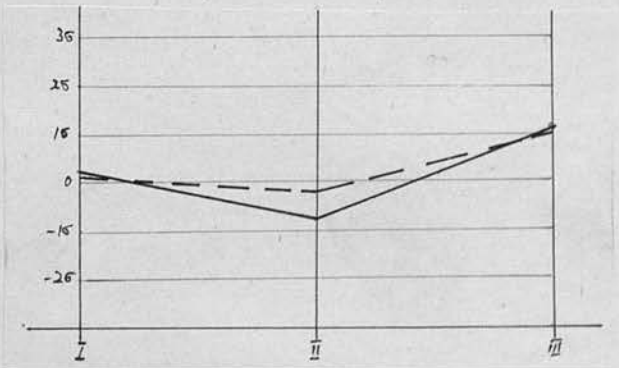

Diagram VIII

Subject No. 100
(Gunnery)

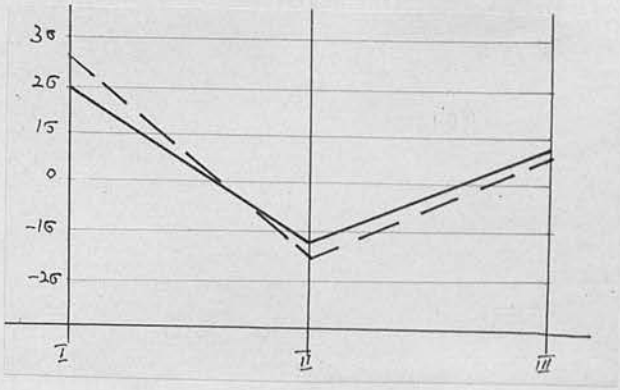

Diagram IX

Subject No. 150
(Infantary)

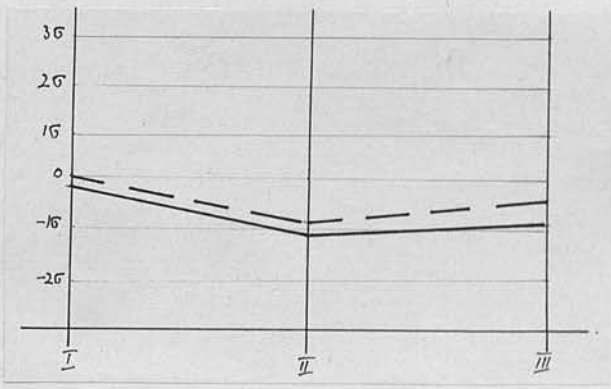

Diagram X

NOTE: The solid line represents observed profile.

The dotted line represents the best profile for this occupation.

The best profiles for various occupations were found by giving such tests to tried soldiers.

Information about a subject's previous experience etc. is contained in The Biographical Questionnaire which is completed by the subject. (See Appendix 5).

VIII. SUMMARY AND CONCLUSION

A battery consisting of three groups of tests, non-verbal tests, verbal tests, and performance tests, was given to 150 Polish soldiers forming a representative sample of the 10,000 tested. The intercorrelations of test scores were analysed by Thurstone's centroid method into three group factors. The centroid factors were rotated, by the method of extended vectors, to an oblique structure. Interpretation of the rotated factors was then discussed. This oblique structure on primary factors was applied as a criterion to find the best weights of the tests on these primary factors. The best predicted scores in primary factors were found and from these certain individual profiles were constructed. These were matched with pattern-profiles previously built for various occupations and the men were so selected.

The following conclusions may be drawn:-

(1) Mental testing is very useful in selecting men for the army, especially when this must be done in a very short time. This shortens the time of training and achieves a better result in a shorter space of time. The weapons of modern warfare are such that on the outbreak of war an efficient organisation must be called into operation very speedily and this can only be achieved by selecting at the outset the "right man for the right job", so that no time is wasted in training unsuitable personnel.

(2) The problem of the demobilised service man is a very real one, many having entered the service with no previous training for civilian life. On their release they are faced with the great problem of fitting themselves for a new kind of life. Here again the results of mental testing can be very useful in advising the service man as/

as to the type of employment for which he is best suited and which will give most satisfaction.

The writer regrets that the follow-up experiment could not be carried out as the Polish Army in Great Britain, owing to the political situation, has been disbanded.

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SECTION II

FURTHER INVESTIGATION OF THE MORAY HOUSE SPACE TEST NO. 1

I. SUMMARY

In 1947 a battery of nine tests, including space tests measuring two and three dimensional spatial ability, was applied to 178 boys aged 11+. Analysis of the correlations indicated the presence of one space factor defined by the two dimensional test and another space factor in which the highest loading was in the three dimensional test.

A follow-up of pupils who had done the whole of Moray House Space Test No. 1 (containing two and three dimensional items) showed that this test appeared to be capable of predicting success in technical subjects in the secondary school.

II. HISTORY OF THE TEST

The need for devising tests to measure spatial ability at the earliest possible age derives emphasis from the recent trend in educational practice to provide technical secondary schools and to ensure that children with an aptitude for technical subjects are selected for these schools. In the vocational guidance field, tests measuring spatial ability have already been used with success by Holliday and Drew in the selection of apprentices.

Work on a paper and pencil space test was initiated in 1944 by Professor Sir Godfrey Thomson and Mr. W. G. Emmett, with the assistance of Mr. J. Gray and Mr. J. Y. Erskine.

Later the development of the test became the responsibility of Mr. J. T. Bain. He incorporated additional items, making 160 items in all, and administered the test to an experimental group of boys. The results were used as a basis for selecting items in the final version of the test (Moray House Space Test 1).

The test falls into two parts, formed conveniently from the sections which deal with two dimensions, and the sections which deal with three dimensions. There are two advantages to be gained from this sub-division (originally suggested by Mr. Emmett to Mr. Mills).

Firstly, more detailed information about the properties of the tests will be obtained, and secondly, two sets of test scores will be made available instead of one, which will materially assist in the factorial study.

The/

The two groups of sections are:-

Part 1 (Two dimensions)

Section		Items
2	Squares	12
3	Letters	10
4	Overlapping shades	8
9	Estimating Areas	12
10	Adding and Subtracting areas	10
	Total	52

Part 2 (Three dimensions)

Section		Items
1	Knots	10
5	Reversed Shapes	7
6	Cubes	12
7	Fitting Shaped Blocks	10
8	Counting Contiguous Blocks	9
	Total	48

In 1946/47 Mills put Moray House Space Test 1 into a battery and gave it to 178 boys at age 11+. The battery consisted of the following tests:-

1. Moray House Intelligence Test (M.H.T.41)
2. Moray House English Test (M.H.E.17)
3. Moray House Mechanical Arithmetic (M.H.A.17)
4. Moray House Problem Arithmetic (M.H.A.17)
5. Raven's Matrices
6. Group Test 70/23
7. Jenkins
8. Moray House Space Test Part 1 (Two dimensions)
9. Moray House Space Test Part 2 (Three dimensions)

The/

The intercorrelations between the test scores of the nine tests are shown in Table I.

TABLE I. MATRIX OF CORRELATIONS

	1	2	3	4	5	6	7	8	9
1	-	794	725	763	643	587	675	668	395
2	794	-	727	725	512	456	569	547	301
3	725	727v	-	759	344	333	411	379	196
4	763	725	759	-	477	493	549	616	349
5	643	512	344	477	-	514	731	622	413
6	587	456	333	493	514	-	601	630	394
7	675	569	411	549	731	601	-	627	470
8	668	547	379	616	622	630	627	-	621
9	395	301	196	349	413	394	470	621	-

NOTE. Decimal points should be understood.

III. THE FACTOR ANALYSIS

The matrix of correlations was analysed by the centroid method, three factors being extracted.

As the question of the number of significant factors was of great importance it was decided to repeat the analysis using Lawley's method of maximum likelihood, which permits a satisfactory test of the significance of the residuals to be made, and therefore of the number of factors. As a result of the analysis by the method of maximum likelihood four significant factors were obtained as has already been reported by Mr. Emmett, the loadings of which are shown in Table II.

TABLE II. FACTOR LOADINGS

	I	II	III	IV
1	.9170	.0745	-.0684	-.0639
2	.8270	.2170	-.0495	-.0297
3	.7557	.5135	.0362	.0903
4	.8398	.2134	.1335	-.0266
5	.7091	-.3642	-.2887	.0542
6	.6476	- .2809	.0104	-.1635
7	.7618	-.3216	-.2362	.1073
8	.7706	-.3820	.2481	-.0988
9	.5011	-.4287	.3460	.2411

The loadings of factors III and IV were tested for significance. The result is shown in Table III.

TABLE III /

TABLE III.

THIRD FACTOR LOADINGS

	Factor loading l	Standard Error σ	$t = \frac{l}{\sigma}$	Significant at 5%
1	-.0684	.0400	1.710	No
2	-.0495	.0534	.927	No
3	.0362	.0396	.914	No
4	.1335	.0497	2.686	Yes
5	-.2887	.0546	5.288	Yes
6	.0104	.0734	.142	No
7	-.2362	.0526	4.490	Yes
8	.2481	.0454	5.465	Yes
9	.3460	.0688	5.029	Yes
	FOURTH FACTOR LOADINGS			
1	-.0639	.0743	.860	No
2	-.0297	.1052	.282	No
3	.0903	.0707	1.277	No
4	-.0266	.0958	.278	No
5	.0542	.0948	.572	No
6	-.1635	.1302	1.256	No
7	.1073	.0866	1.239	No
8	-.0988	.0698	1.415	No
9	.2411	.0768	3.139	Yes

From this table we see that tests 4, 5, 7, 8 and 9 have significant loadings in factor III, and in factor IV only test 9 (space test three dimensional part) has significant loading.

When/

When the factors II, III and IV are considered as bipolar the tests are clustered as follows:-

FACTOR II

Positive Loadings

3.	Mechanical Arithmetic	.5135	A
2.	English	.2170	
4.	Problem Arithmetic	.2134	
1.	M.H.T.41	.0745	

Negative Loadings

6.	N.I.I.P.	-.2809	B
7.	Jenkins	-.3216	
5.	Raven	-.3642	
8.	Two dimensions	-.3820	C
9.	Three dimensions	-.4287	

FACTOR III

Positive Loadings

9.	Three dimensions	.3460	C
8.	Two dimensions	.2481	
4.	Problem Arithmetic	.1335	A
3.	Mechanical Arithmetic	.0362	
6.	N.I.I.P.	.0104	B

Negative Loadings

2.	English	-.0495	A
1.	M.H.T.41	-.0684	
7.	Jenkins	-.2362	B
5.	Raven	-.2887	

FACTOR IV

Positive Loadings

9.	Three dimensions	.2411	C
7.	Jenkins	.1073	B
3.	Mechanical Arithmetic	.0903	A
5.	Raven	.0542	B

Negative Loadings

4.	Problem Arithmetic	-.0266	A
2.	English	-.0297	
1.	M.H.T.41	-.0639	
8.	Two dimensions	-.0988	C
6.	N.I.I.P.	-.1635	B

The clusters break up in the fourth factor and it is remarkable that in the third factor the Matrix and Space three-dimension tests diverge most.

The clustering of the tests in factors II and III suggests three oblique factors. It was therefore decided to rotate the first three factors by the method of extended vectors.

IV. EXTENDED VECTOR ANALYSIS OF THE FIRST THREE FACTORS

The first three factors were rotated to a simple structure by the method of extended vectors (Thurstone). The projections of all test vectors on factor I are made equal to unity by dividing each row by its first member. The extended matrix so derived is shown in Table IV.

TABLE IV. EXTENDED LOADINGS

	I_e	II_e	III_e
1	1.0000	.0595	-.0887
2	1.0000	.2405	-.0606
3	1.0000	.6368	.0913
4	1.0000	.2253	.1555
5	1.0000	-.5166	-.3922
6	1.0000	-.4603	-.0571
7	1.0000	-.4324	-.2833
8	1.0000	-.5220	.2450
9	1.0000	-.9407	.7529

When these values are plotted a pattern of dots is obtained on a plane II_eIII_e, which is at unit distance along axis I_e and at right angles to it. The columns II_e and III_e of this matrix are the coordinates of the dots on the above-mentioned plane.

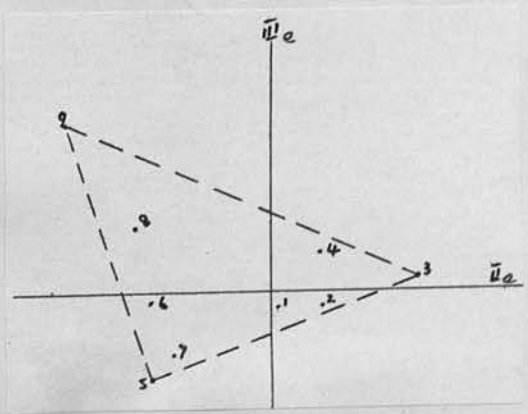

Diagram I - Configuration of Test Points.

Having plotted the points shown in the diagram our next task is to draw the lines shown. These are drawn in such a way that they pass through or near to as many points as possible, producing a triangle.

It is clear, however, that the lines are not equally well defined. That running from test-point 3 (Mechanical Arithmetic) to the group of tests 5, 6 and 7 (Raven, Group Test 70/23, and Jenkins) is well defined, passing on the way through the Moray House Intelligence Test and near English and Problem Arithmetic. But the other two lines of the triangle depend on test-point 9 (Three-dimensional diagrams), that from 3 to 9 entirely so, that from 5 to 9 partly supported by test-point 8. Clearly a new experiment is indicated (and is under way) with an enlarged battery containing tests which may help to define these two sides of the triangle.

The equations of the lines were found, of which the coefficients, after being normalised, form the rotating matrix, by which the original matrix F_o of factor loadings must be post-multiplied in order to get simple structure. The oblique structure thus obtained is shown in Table V.

TABLE V. OBLIQUE SIMPLE STRUCTURE

	I	II	III
1	.3399	.0519	.5044
2	.2320	.0108	.5839
3	.0000	.0000	.8125
4	.0800	.1818	.6350
5	.5889	.0000	.0000
6	.3414	.1817	.0921
7	.5400	.0502	.0743
8	.2207	.4415	.1392
9	.0000	.5947	.0000

The three factors are defined by:

Factor I Jenkins, Group Test 70/23, and Raven Matrix

Factor II Three-dimensional Space Test

Factor III Mechanical Arithmetic

and their angular positions are:

	I	II	III
I		47^{o}	63^{o}
II			75^{o}
III			

A general factor, in the sense of Spearman's G does not appear in such solutions, although in certain types of problems a general factor may subsequently be extracted as a second-order factor.

The tests 2 and 4 are explained in bigger part by the factor III and in smaller part by the factor I.

The test 1 is equally explained by factors I and III. The test 8 is mostly explained by factor II and has a small loading in factor I.

In an attempt to identify a fourth factor, diagrams were drawn and hyperplanes investigated, but any definable fourth factor appears to fall very close to one or other of the former three factors. The fourth dimension of the common factor space is in fact not well defined. The attempt with this battery to define a fourth factor was therefore abandoned.

V. THE PREDICTIVE VALUE OF MORAY HOUSE SPACE TEST No. 1 FOR ATTAINMENT IN THE SECONDARY SCHOOL

In January 1947 a complete year group of 11+ children in an industrial town was given Moray House Intelligence, English, Arithmetic and Space Tests. On the result of these tests children were allocated to Grammar and Modern Secondary Schools.

Of the Grammar School boys 71 were assessed at the end of their first and second years in the Grammar School by the teachers in Geometry, Science and Art. As expected the range of ability was very small with a standard deviation of I.Q. of 6.32 compared with that in the population of 15.

The Grammar School marks in the three subjects were then correlated with the four variables in the admission examination; these correlations are recorded in Table VI.

TABLE VI. CORRELATION COEFFICIENTS (GRAMMAR SCHOOL)

		AQ	EQ	IQ	Sp.	Mult. Corr.	Corr. bet. 1st & 2nd years
Geometry	1st	.3186	.1463	.5131	.4922	.6162	.5884
	2nd	.4531	.2101	.5457	.5921	.7040	
Science	1st	.3515	.1142	.4336	.4335	.5591	.4288
	2nd	.1197	.0888	.3567	.3237	.4078	
Art	1st	-.0014	-.0339	.1963	.4202	.4679	.2824
	2nd	-.1445	-.0759	.2079	.3186	.4544	

It will be seen that the correlations of the Space Test with each school subject are significant and in the cases of Geometry and Art for the second year are higher than those of any other of the variables.

It is interesting to note that the multiple correlations, which represent the correlation of the battery as a whole with the/

the optimum weight assigned to each test, are high for such a stringently selected sample and will probably exceed 0.8 when referred to a representative sample of the population.

Turning now to the regression coefficients given in Table VII, these show the relative predictive values of each test when competing with others in the same battery.

TABLE VII. REGRESSION COEFFICIENTS (GRAMMAR SCHOOL)

		AQ	EQ	IQ	Sp.
Geometry	1st	.1353	-.1639	.4093	.3058
	2nd	.2580	-.1355	.3220	.3910
Science	1st	.2192	-.1832	.3360	.2533
	2nd	-.0306	-.0860	.3050	.2120
Art	1st	-.1504	-.1335	.1087	.4590
	2nd	-.2802	-.1448	.2126	.3477

It is seen that the regression coefficients of the Space Test are all significant (except second year Science). Indeed, the Space Test and IQ do all the work in predicting the school subjects.

The second follow-up was carried out on 44 boys selected from the same year group as previously mentioned who had proceeded to a Modern Secondary School and were assessed by the teachers after one year and after two years' work in Technical Drawing, Science, Woodwork and Metalwork. This group was also a highly selected group, with a standard deviation of IQ of 8.50.

The correlations after first and second years are given in Table VIII.

TABLE VIII/

TABLE VIII. CORRELATION COEFFICIENTS (MODERN SCHOOL)

		AQ	EQ	IQ	Sp.	Mult. Corr.	Corr. bet. 1st & 2nd years
Techn. Drawing	1st	.6487	.4574	.6173	.5527	.7108	.5925
	2nd	.4084	.2830	.3697	.4176	.4779	
Science	1st	.4305	.2956	.5525	.4593	.5938	.3114
	2nd	-.0100	-.0182	.2188	.2244	.3993	
Woodwork	1st	.0215	-.1097	.1560	.2828	.4441	.3165
	2nd	-.0767	-.1651	-.1162	-.0508	.1719	
Metalwork	1st	.3696	.0768	.1238	.1009	.4180	.3875
	2nd	.3804	.2110	.3323	.4039	.4637	

The correlations of the Space Test with Technical Drawing are significant, and with Science for the first year the correlation is high and second only to that of IQ. The correlation of the Space Test with Metalwork after two years is higher than that of any other variable. The resulting regression coefficients, given in Table IX for the Space Test are all positive and though none is significant on account of the small number of cases employed they give encouraging evidence that the inclusion of the Space Test in the predicting battery is well justified.

TABLE IX/

TABLE IX. REGRESSION COEFFICIENTS (MODERN SCHOOL)

		AQ	EQ	IQ	Sp.
Techn. Drawing	1st	.4272	-.0713	.2275	.2176
	2nd	.2827	-.0271	-.0094	.2972
Science	1st	.1384	-.2667	.5593	.1369
	2nd	-.2470	-.3437	.5350	.1500
Woodwork	1st	-.0981	-.4693	.3307	.3406
	2nd	.0299	-.1788	-.0448	.0571
Metalwork	1st	.5516	-.1339	-.1535	.0014
	2nd	.2925	-.1258	.0031	.3202

The drop of multiple correlation coefficients is possibly due to: (a) the predictive value of the battery decreasing with the passage of time; (b) this battery not being the best one for predicting all of the subjects. The recorded correlation coefficients between marks of the first and second years for each school subject are low. This is probably on account of: (a) children not being assessed by the same teacher, or (b) a change in the content of a subject.

In general the results from these follow-ups should be interpreted with caution because of the small number involved, but they may be regarded as indicative of some promise for the prediction of spatial ability at the age of 11+.

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APPENDIX 1

RAW SCORES (150 cases)

	Picture tests			Verbal tests			Performance tests		
	1	2	3	4	5	6	7	8	9
1	49	18	34	12	15	8	64	12	66
2	48	20	32	13	14	10	61	14	64
3	45	19	32	14	15	10	61	13	65
4	44	21	31	14	18	11	60	15	67
5	44	17	31	17	18	11	59	11	63
6	44	19	25	8	15	15	58	14	39
7	44	21	24	9	14	16	55	15	40
8	44	20	24	8	13	17	56	16	38
9	44	18	23	7	16	17	57	13	36
10	44	22	24	8	17	20	54	12	37
1	43	15	32	15	11	20	63	18	37
2	43	14	33	18	10	17	62	19	38
3	42	16	35	16	9	16	64	20	39
4	41	17	32	20	7	17	66	17	41
5	41	18	33	20	8	15	65	16	40
6	41	14	30	21	52	23	43	17	34
7	41	19	29	23	51	26	40	16	33
8	41	17	29	22	53	25	40	15	32
9	41	18	27	24	55	24	39	18	35
20	41	17	30	25	54	27	38	19	36
1	39	25	32	20	28	27	48	18	33
2	37	23	34	20	29	29	49	19	32
3	37	24	32	20	27	28	47	17	31
4	36	20	31	20	25	30	50	20	34
5	36	23	31	20	26	26	46	16	35
6	36	26	22	28	28	27	63	15	37
7	36	25	23	29	27	24	60	14	36
8	35	24	23	32	31	25	60	13	35
9	34	23	24	31	30	26	59	16	33
30	34	22	23	30	29	23	58	17	34
1	34	29	37	11	34	24	62	13	32
2	34	29	36	9	33	25	63	12	31
3	34	28	25	10	35	26	64	11	30
4	34	28	35	8	37	28	61	9	29
5	34	26	32	7	36	27	60	10	28
6	34	24	39	17	30	25	65	10	21
7	33	25	39	17	33	23	64	12	21
8	32	26	38	17	31	24	63	13	21
9	31	27	38	17	29	22	66	11	21
40	30	28	37	17	32	26	67	9	21
1	30	34	34	12	31	27	66	10	19
2	30	33	34	11	34	26	70	11	20
3	30	33	34	10	32	28	67	9	18
4	30	32	34	8	33	30	69	7	16
5	30	33	34	9	30	29	68	8	17
6	30	32	34	31	40	53	81	9	18
7	28	31	33	35	38	54	82	7	17
8	28	31	32	34	41	52	78	8	19
9	27	30	30	33	39	51	80	11	21
50	27	31	32	32	37	50	79	10	21
1	27	29	38	33	39	19	75	13	23
2	27	31	39	34	40	22	72	12	23
3	27	30	40	32	42	20	74	14	23
4	27	30	42	30	43	21	73	16	23
5	27	30	41	31	41	18	76	15	23

	Picture tests			Verbal tests			Performance tests		
	1	2	3	4	5	6	7	8	9
56	26	23	39	20	10	16	65	12	52
7	26	25	30	21	9	13	66	9	51
8	26	24	37	17	7	14	64	10	50
9	26	20	36	19	6	15	62	11	49
60	26	23	35	18	8	12	63	8	48
1	25	23	37	36	35	45	73	10	51
2	25	23	38	39	36	48	74	10	52
3	25	21	41	38	39	46	75	10	53
4	25	20	40	35	37	47	77	10	55
5	25	18	38	37	38	44	76	10	54
6	24	16	35	39	48	29	77	12	30
7	24	18	35	39	44	32	77	11	29
8	24	19	34	39	46	30	77	10	28
9	24	17	34	39	43	31	77	8	26
70	24	20	33	39	47	28	77	9	27
1	23	15	26	39	40	53	80	10	30
2	23	13	26	29	41	50	81	10	29
3	23	14	26	27	44	51	82	10	26
4	23	12	26	28	42	52	84	10	27
5	23	16	26	26	43	49	83	10	28
6	23	14	33	25	10	10	77	15	28
7	22	14	34	20	7	9	78	14	27
8	22	13	35	19	6	8	76	16	29
9	22	11	34	21	9	7	74	18	25
80	21	13	35	18	8	6	75	17	26
1	21	14	19	9	17	32	78	17	21
2	21	14	14	8	19	33	79	18	22
3	21	13	16	8	18	31	80	16	23
4	21	18	16	8	21	30	80	14	24
5	21	18	15	7	20	39	81	15	25
6	21	17	20	33	43	42	70	15	23
7	20	16	22	32	41	45	74	17	25
8	20	15	22	31	42	43	72	16	24
9	20	14	21	29	40	44	73	19	22
90	19	13	25	30	44	41	71	18	21
1	19	13	20	12	25	42	81	19	49
2	19	16	21	11	26	43	85	18	50
3	19	15	19	9	27	41	83	17	51
4	19	17	18	10	29	39	84	16	52
5	19	14	17	8	28	40	82	15	48
6	19	16	17	33	50	39	84	15	42
7	18	13	15	34	54	42	85	16	39
8	18	14	16	35	55	40	86	18	40
9	18	12	14	37	50	41	88	17	41
100	17	15	13	36	51	38	87	19	38
1	17	10	22	26	7	8	64	14	21
2	17	10	24	24	10	9	65	13	22
3	17	9	23	25	8	7	66	16	23
4	17	8	23	22	6	5	67	17	19
5	17	8	23	23	9	6	68	15	20
6	17	11	23	13	25	7	36	15	47
7	16	11	23	14	25	6	37	17	49
8	16	10	23	12	25	8	35	13	48

	Picture tests			Verbal tests			Performance tests		
	1	2	3	4	5	6	7	8	9
109	16	9	22	10	25	5	34	14	50
110	15	9	24	11	25	9	33	16	46
1	15	14	19	12	25	10	46	11	30
2	15	12	15	15	25	11	47	8	28
3	15	12	17	14	26	12	45	9	29
4	15	13	18	13	24	9	43	10	27
5	15	9	16	11	25	8	44	7	26
6	15	14	20	15	20	32	32	10	16
7	14	14	21	16	17	34	34	13	13
8	14	14	25	14	18	36	33	12	14
9	14	11	22	12	19	35	35	11	15
120	13	12	22	13	16	33	31	14	12
1	13	16	16	12	20	37	31	7	15
2	13	12	14	14	18	39	32	6	12
3	13	14	15	15	19	38	30	8	13
4	13	13	12	16	16	40	28	10	14
5	13	15	13	13	17	36	29	9	11
6	13	12	19	13	17	38	31	9	14
7	12	13	20	15	18	37	34	10	16
8	12	12	21	16	19	39	32	8	15
9	12	14	23	14	16	40	33	7	17
130	11	9	22	17	15	36	30	6	13
1	11	9	22	17	12	13	30	6	12
2	11	8	21	18	13	14	30	7	15
3	11	7	20	16	11	12	30	5	13
4	11	5	19	15	9	11	30	9	14
5	11	6	18	14	10	10	30	8	16
6	11	14	19	15	10	17	53	9	20
7	10	16	16	19	7	19	54	8	18
8	10	18	16	17	8	18	52	10	19
9	10	18	15	18	9	20	50	12	17
140	9	14	14	16	6	16	51	11	21
1	9	10	14	15	23	17	32	9	15
2	9	8	13	16	22	20	31	8	16
3	9	8	12	20	24	19	30	6	16
4	9	8	10	18	21	18	28	7	17
5	9	6	11	20	20	21	29	5	16
6	9	8	20	20	22	21	35	5	16
7	8	10	19	15	20	19	34	6	15
8	8	7	18	20	21	20	36	9	15
9	8	7	16	18	19	17	38	8	16
150	7	8	17	16	23	18	37	7	18

CENTROID ANALYSIS (CALCULATION)

MATRIX OF INTERCORRELATIONS OF 9 TESTS (150 CASES):- R

	1	2	3	4	5	6	7	8	9
1	(.6316)	.6020	.6316	.0197	.1966	-.0729	.3279	.4468	.5207
2	.6020	(.6833)	.6833	.1508	.3554	.2347	.4544	.0360	.1622
3	.6316	.6833	(.6833)	.2774	.1942	-.0835	.3811	.1177	.3799
4	.0197	.1508	.2774	(.5616)	.5616	.3932	.4418	.0648	.0809
5	.1966	.3554	.1942	.5616	(.5830)	.5830	.3707	-.0264	.1002
6	-.0729	.2347	-.0835	.3932	.5830	(.5830)	.3573	-.0562	-.1030
7	.3279	.4544	.3811	.4418	.3707	.3573	(.5340)	.5340	.4462
8	.4468	.0360	.1177	.0648	-.0264	-.0562	.5340	(.5340)	.4697
9	.5207	.1622	.3799	.0809	.1002	-.1030	.4462	.4697	(.5207)
Sum	3.3040	3.3621	3.2650	2.5518	2.9183	1.8356	3.8474	2.1204	2.5775
1st factor load's	.6507	.6622	.6430	.5026	.5747	.3615	.7577	.4176	.5076

$$G.T. = 25.7821 \quad (25.7821)^{-\frac{1}{2}} = .196943$$

MATRIX OF CORRELATIONS DUE TO FACTOR I:- R_{f_1}

	1	2	3	4	5	6	7	8	9
1	(.4234)	.4309	.4184	.3270	.3740	.2352	.4930	.2717	.3303
2		(.4385)	.4259	.3328	.3806	.2394	.5017	.2765	.3361
3			(.4134)	.3232	.3695	.2324	.4872	.2685	.3264
4				(.2526)	.2888	.1817	.3808	.2099	.2551
5					(.3303)	.2078	.4355	.2400	.2917
6						(.1307)	.2739	.1510	.1835
7							(.5741)	.3164	.3846
8								(.1744)	.2120
9									(.2577)

NOTE. Numbers in brackets are the squares of corresponding first factor loadings.

MATRIX OF FIRST FACTOR RESIDUALS:- $R-R_{f_1}$

(Signs changed in tests 4,5,6 & 7)

	1	2	3	4	5	6	7	8	9
1	(.3081)	.1711	.2132	.3073	.1774	.3081	.1651	.1751	.1904
2	.1711	(.2574)	.2574	.1820	.0252	.0047	.0473	-.2405	-.1739
3	.2132	.2574	(.3159)	.0458	.1753	.3159	.1061	-.1508	.0535
4	.3073	.1820	.0458	(.3073)	.2728	.2115	.0610	.1451	.1742
5	.1774	.0252	.1753	.2728	(.3752)	.3752	-.0648	.2664	.1915
6	.3081	.0047	.3159	.2115	.3752	(.3752)	.0834	.2072	.2865
7	.1651	.0473	.1061	.0610	-.0648	.0834	(.2176)	-.2176	-.0616
8	.1751	-.2405	-.1508	.1451	.2664	.2072	-.2176	(.2664)	.2577
9	.1904	-.1739	.0535	.1742	.1915	.2865	-.0616	.2577	(.2865)
Sum	2.0158	.5307	1.3323	1.7070	1.7942	2.1677	.3365	.7090	1.2048
2nd factor load's	.5869	.1545	.3879	.4970	.5223	.6310	.0980	.2064	.3508

G.T. = 11.7980 $\quad (11.7980)^{-\frac{1}{2}} = .291137$

MATRIX OF CORRELATIONS DUE TO FACTOR II:- R_{f_2}

	1	2	3	4	5	6	7	8	9
1	(.3445)	.0907	.2277	.2917	.3065	.3703	.0575	.1211	.2059
2		(.0239)	.0599	.0768	.0807	.0975	.0151	.0319	.0542
3			(.1505)	.1928	.2026	.2448	.0380	.0801	.1361
4				(.2470)	.2596	.3136	.0487	.1026	.1743
5					(.2728)	.3296	.0512	.1078	.1832
6						(.3982)	.0618	.1302	.2214
7							(.0096)	.0202	.0344
8								(.0426)	.0724
9									(.1231)

NOTE. Numbers in brackets are the squares of corresponding second factor loadings.

MATRIX OF SECOND FACTOR RESIDUALS:- $R-R_{f_1}-R_{f_2}$

(Signs changed in tests 5,6,8 & 9)

	1	2	3	4	5	6	7	8	9
1	(.1291)	.0804	-.0145	.0156	.1291	.0622	.1076	-.0540	.0155
2	.0804	(.2724)	.1975	.1052	.0555	.0928	.0322	.2724	.2281
3	.0145	.1975	(.2309)	-.1470	.0273	-.0711	.0681	.2309	.0826
4	.0156	.1052	.1470	(.1470)	-.0132	.1021	.0123	-.0425	.0001
5	.1291	.0555	.0273	-.0132	(.1586)	.0456	.1160	.1586	.0083
6	.0622	.0928	-.0711	.1021	.0456	(.1021)	-.0216	.0770	.0651
7	.1076	.0322	.0681	.0123	.1160	-.0216	(.2378)	.2378	.0960
8	-.0540	.2724	.2309	-.0425	.1586	.0770	.2378	(.2724)	.1853
9	.0155	.2281	.0826	.0001	.0083	.0651	.0960	.1853	(.2281)
Sum	.4710	1.3365	.6047	.1796	.6858	.4542	.8862	1.3379	.9091
3rd factor load's	.1798	.5101	.2308	.0685	.2617	.1734	.3382	.5106	.3470

G.T. = 6.8650 $\quad (6.8650)^{-\frac{1}{2}} = .381663$

MATRIX OF CORRELATIONS DUE TO FACTOR III:- R_{f_3}

	1	2	3	4	5	6	7	8	9
1	(.0323)	.0917	.0415	.0123	.0471	.0312	.0608	.0918	.0624
2		(.2602)	.1177	.0349	.1335	.0885	.1725	.2605	.1770
3			(.0533)	.0158	.0604	.0400	.0781	.1178	.0801
4				(.0047)	.0179	.0119	.0232	.0350	.0238
5					(.0685)	.0454	.0885	.1336	.0908
6						(.0301)	.0586	.0885	.0602
7							(.1144)	.1727	.1174
8								(.2607)	.1772
9									(.1204)

NOTE. Numbers in brackets are the squares of corresponding third factor loadings.

MATRIX OF THIRD FACTOR RESIDUALS:- $R-R_{f_1}-R_{f_2}-R_{f_3}$

	1	2	3	4	5	6	7	8	9
1	(.1458)	-.0113	-.0560	.0033	.0820	.0310	.0468	-.1458	-.0469
2	-.0113	(.1403)	.0798	.0703	-.0780	.0043	-.1403	.0119	.0511
3	-.0560	.0798	(.1628)	-.1628	-.0331	-.1111	-.0100	.1131	.0025
4	.0033	.0703	-.1628	(.1628)	-.0311	.0902	-.0109	-.0775	-.0237
5	.0820	-.0780	-.0331	-.0311	(.0825)	.0002	.0275	.0250	-.0825
6	.0310	.0043	-.1111	.0902	.0002	(.1111)	-.0802	-.0115	.0049
7	.0460	-.1403	-.0100	-.0109	.0275	-.0802	(.1433)	.0651	-.0214
8	-.1458	.0119	.1131	-.0775	.0250	-.0115	.0651	(.1458)	.0081
9	-.0469	.0511	.0025	-.0237	-.0825	.0049	-.0214	.0081	(.0825)

TEST FOR SIGNIFICANCE OF THIRD FACTOR RESIDUALS

The third factor residuals were tested for significance by Quin McNemar's method. It was found that the ratio between the standard deviation of the last residuals (.0104) and departure from unity of the mean of communalities (after three factors) is less than $1/\sqrt{N}$, (.082), where N is the number of cases in the sample. It suggests that the magnitude of the residuals are such that their departure from zero may be considered as due to chance sampling errors in the original intercorrelations.

CALCULATION OF A RECIPROCALOOF A MATRIX USING PIVOTAL CONDENSATION METHOD (AITKEN)

						Check
K =	1.0000	.1071	.0909	-1		.1980
	.1071	1.0000	-.1560		-1	-.0489
	.0909	-.1560	1.0000			-.0651
	1					1.0000
		1				1.0000
			1			1.0000
.9885 ÷		.9885	-.1657	.1071	-1	-.0701
		1	-.1676	.1083	-1.0116	-.0709
		-.1657	.9917	.0909	-1	-.0831
		-.1071	-.0909	1		.8020
		1				1.0000
			1			1.0000
.9642 ÷			.9642	.1088	-.1676 -1	-.0948
			1	.1128	-.1738 -1.0373	-.0983
			-.1088	1.0116	-.1083	.7944
			.1676	-.1083	1.0116	1.0709
			1			1.0000
		K^{-1} =		1.0239	-.1272 -.1128	.7837
				-.1272	1.0407 .1738	1.0874
				-.1128	.1738 1.0373	1.0983

As a check:- $K \times K^{-1} = I$ = unit matrix

CALCULATION OF THE BEST PREDICTED SCORES IN FACTORS

Subject No. 1

Test	z	zb_1	zb_2	zb_3
1	2.2852	-1.0510	1.4984	.6415
2	.0632	.0261	.0162	-.0302
3	.9427	-.0633	.2106	.1409
4	-.8605	-.2531	.0933	-.0224
5	-.8269	-.3811	.0290	.1194
6	-1.3623	-.3949	.0375	.1343
7	.2932	-.0416	-.0071	.2010
8	-.1227	-.0250	.0230	-.0120
9	2.6418	.0222	-.0444	.5091
Best predicted scores		-2.1617	1.8565	1.6816

Subject No. 71

Test	z	zb_1	zb_2	zb_3
1	-.0886	.0407	-.0581	-.0249
2	-.3580	-.1476	-.0915	.1709
3	.0035	-.0002	.0008	.0005
4	2.0366	.5990	-.2208	.0530
5	1.0156	.4681	-.0356	-.1467
6	2.0172	.5848	-.0555	-.1989
7	1.1652	-.1653	-.0283	.7990
8	-.6449	-.1312	.1208	-.0631
9	-.0222	-.0002	.0004	-.0043
Best predicted scores		1.2481	-.3678	.5855

Subject No. 92

Test	z	zb_1	zb_2	zb_3
1	-.4538	.2087	-.2976	-.1274
2	-.2176	-.0897	-.0556	.1039
3	-.5835	.0395	-.1304	-.0872
4	-.9678	-.2846	.1049	-.0252
5	-.0162	-.0075	.0006	.0023
6	1.2662	.3671	-.0348	-.1248
7	1.4377	-.2040	-.0349	.9858
8	1.4439	.2937	-.2704	.1412
9	1.4578	.0122	-.0245	.2809
Best predicted scores		.3351	-.7427	1.1495

Subject No. 100

Test	z	zb_1	zb_2	zb_3
1	-.6364	.2927	-.4173	-.1786
2	-.3580	-.1476	-.0915	.1709
3	-1.5227	.1023	-.3402	-.2276
4	1.7147	.5043	-.1859	.0446
5	1.8263	.8417	-.0641	-.2637
6	.8907	.2582	-.0245	-.0878
7	1.5467	-.2185	-.0376	1.0606
8	1.7050	.3468	-.3193	.1667
9	.5698	.0048	-.0096	.1098
Best predicted scores		1.9837	-1.4900	.7949

Subject No. 150

Test	z	zb_1	zb_2	zb_3
1	-1.5494	.7126	-1.0159	-.4349
2	-1.3408	-.5527	-.3428	.6402
3	-1.0531	.0708	-.2353	-.1574
4	-.4313	-.1268	.0468	-.0112
5	-.2373	-.1094	.0083	.0343
6	-.6113	-.1772	.0168	.0603
7	-1.1783	.1672	.0286	-.8080
8	-1.4282	-.2905	.2675	-.1397
9	-.9102	-.0076	.0153	-.1754
Best predicted scores		-.3136	-1.2107	-.9918

where $z = (x-\bar{x})/\sigma$ and x = raw score, $\bar{x}$ = mean raw score,

b = regression coefficient.

THE BIOGRAPHICAL QUESTIONNAIRE
(given in Polish)

S.F. Sheet 100A (July 43) **QUALIFICATION FORM**

CONFIDENTIAL

Do NOT write in this margin.

A. IDENTIFICATION PARTICULARS

1.—P.T.C/W. No. ..... 2.—Army No. .....

3.—Surname.....

4.—Initials ..... 5.—Date of Birth.....

6.—Date of Intake..... 7.—Nature of Engagement.....

9.—Army Trade, if any .....

10.—Nationality ..... 10a.—County Connection.....

8. Arm .....

11. I.R.C. .....

12. E.S. .....

B. EDUCATION

13.—Age on leaving last full-time school..... 14.—Standard/Class.....

15.—Matriculation or School Certificate..... 16.—Higher School Certificate.....

17.—H.S.C. Subjects.....

18.—University..... 19.—Subjects..... 20.—Class.....

21.—Subjects taken at professional courses, including technical courses and evening classes :—

Subject	No. of Sessions	Where Taken

C EMPLOYMENT 22.—

Employer	Your job	How long	Usual wages

23.—Which of these has been your *main* job ?.....

24.—What exactly did you do ?.....

.....

25.—What job do you intend to do after the war ?.....

.....

26.—What is or was your father's job ?.....

D. OTHER EXPERIENCE

27.—Underline whichever of these you can drive :—

Car Motor-cycle P.S.V. Heavy Goods Motor Boat

28.—How long have you held a driving licence ? From 19..... to 19.....

29.—Underline the type of vehicle for which you have passed a driving test :—

Car Motor-cycle P.S.V. Heavy Goods

30.—Underline any of the following of which you have had experience, either in your job or in your spare time :—

(*a*) Metalwork (*e*) Car repairs (*h*) Typing (*l*) Cooking
(*b*) Woodwork (*f*) Using precision tools (*j*) Shorthand (*m*) Morse Code
(*c*) Electrical repairs (*g*) Handling boats (*k*) First Aid (*n*) Photography
(*d*) Radio repairs

31.—Previous experience in H.M. Forces (incl. Home Guard), Civil Defence, Training or Cadet Corps :—

Service	Approx. Dates	Rank	Nature of Duties

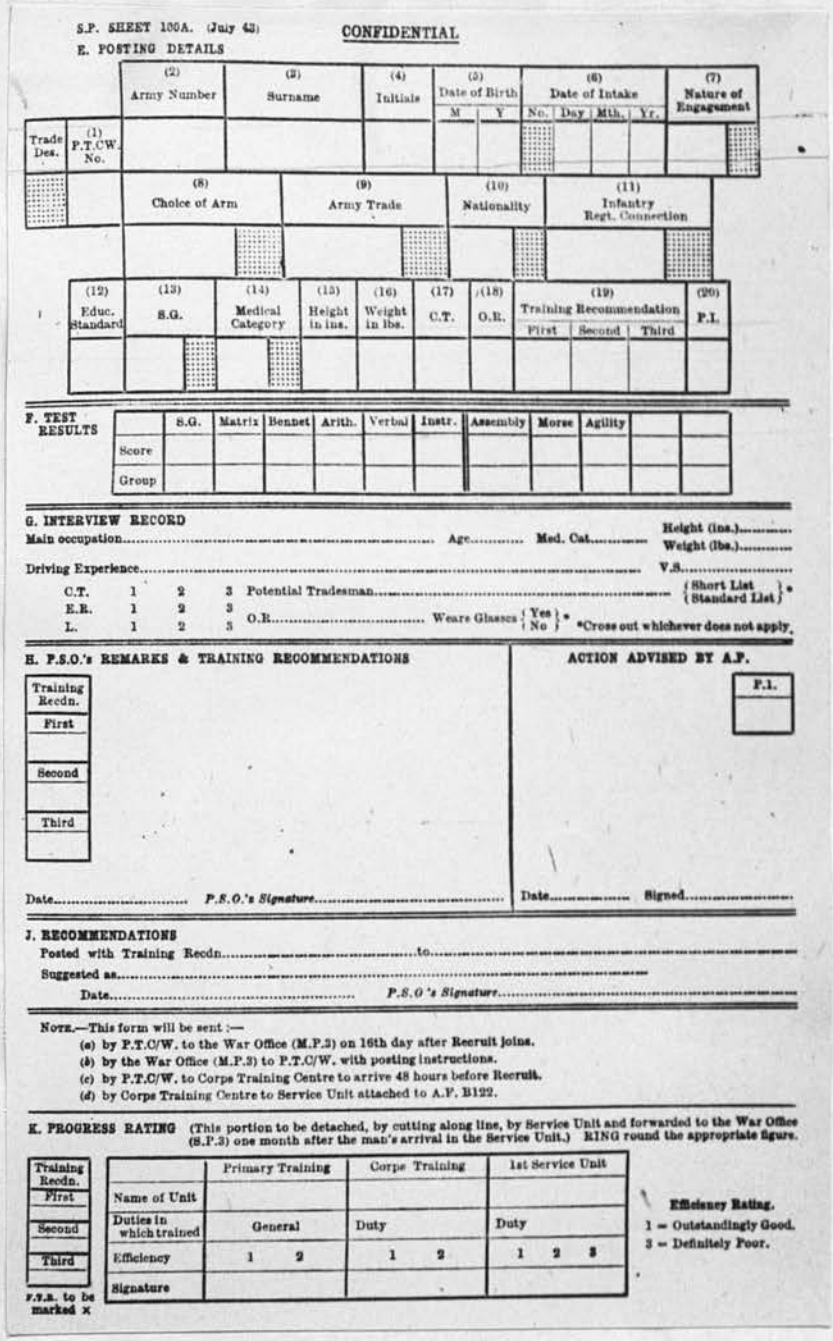
32.—Were you a member of any boys' organisation ?

Which ?..... How long ?.....

Date..... *Signature*.....

Front page of Army Qualification Form

THE BIOGRAPHICAL QUESTIONNAIRE
(given in Polish)



S.P. SHEET 100A. (July 43) CONFIDENTIAL

E. POSTING DETAILS

Trade Des.	(1) P.T.C.W. No.	(2) Army Number	(3) Surname	(4) Initials	(5) Date of Birth M	(5) Date of Birth Y	(6) Date of Intake No.	(6) Date of Intake Day	(6) Date of Intake Mth.	(6) Date of Intake Yr.	(7) Nature of Engagement

(8) Choice of Arm		(9) Army Trade		(10) Nationality		(11) Infantry Regt. Connection	

(12) Educ. Standard	(13) S.G.		(14) Medical Category		(15) Height in ins.	(16) Weight in lbs.	(17) C.T.	(18) O.R.	(19) Training Recommendation First	(19) Second	(19) Third	(20) P.I.

F. TEST RESULTS

	S.G.	Matrix	Bennet	Arith.	Verbal	Instr.	Assembly	Morse	Agility		
Score											
Group											

G. INTERVIEW RECORD

Main occupation..... Age..... Med. Cat..... Height (ins.)..... Weight (lbs.).....

Driving Experience..... V.S.....

C.T. 1 2 3 Potential Tradesman..... {Short List / Standard List} *

E.R. 1 2 3

L. 1 2 3 O.R..... Wears Glasses {Yes / No} *

*Cross out whichever does not apply.

H. P.S.O.'s REMARKS & TRAINING RECOMMENDATIONS

Training Recdn.
First
Second
Third

Date..... *P.S.O.'s Signature*.....

ACTION ADVISED BY A.F.

P.I.

Date..... Signed.....

J. RECOMMENDATIONS

Posted with Training Recdn..... to.....

Suggested as.....

Date..... *P.S.O.'s Signature*.....

NOTE.—This form will be sent :—

(a) by P.T.C/W. to the War Office (M.P.3) on 16th day after Recruit joins.
(b) by the War Office (M.P.3) to P.T.C/W. with posting instructions.
(c) by P.T.C/W. to Corps Training Centre to arrive 48 hours before Recruit.
(d) by Corps Training Centre to Service Unit attached to A.F. B122.

K. PROGRESS RATING (This portion to be detached, by cutting along line, by Service Unit and forwarded to the War Office (S.P.3) one month after the man's arrival in the Service Unit.) RING round the appropriate figure.

Training Recdn.		Primary Training	Corps Training	1st Service Unit
First	Name of Unit			
Second	Duties in which trained	General	Duty	Duty
Third	Efficiency	1 2	1 2	1 2 3
	Signature			

P.T.C. to be marked ×

Efficiency Rating.
1 = Outstandingly Good.
3 = Definitely Poor.

Back page of Army Qualification Form

SPECIMEN COPY OF MORAY HOUSE SPACE TEST NO. 1.

DO NOT TURN OVER OR OPEN THIS BOOK UNTIL YOU ARE TOLD

No. 73

SPACE TEST I

Fill in the following particulars at once :—

To-day's Date.....19.....

Your Surname (in capitals).....

Your Christian Name(s).....

Your Sex (Boy or Girl).....

Name of your School.....

Class you are in.....

Your Age.....Years

Date of your Birthday.....

Not to be filled in by Pupil.

Page	Score
1.	
3.	
5.	
6.	
7.	
8.	
9.	
TOTAL	

Signature of Marker:

Age in years and completed months

y. m.

Space Quotient:

Read the following carefully :—

1. When you are told to begin, turn over to page 1 and start working at once. At the end of each page, follow the instructions given at the bottom.
2. The test is in five sections. You will be told how much time is allowed for each section.
3. Each time you are told to stop, STOP WORKING AT ONCE.
4. Work as quickly and as carefully as you can.
5. If after trying a question you find you cannot do it, do not waste time but go on to the next.
6. Make any alterations in your answers clearly.
7. **Ask no questions at all.**

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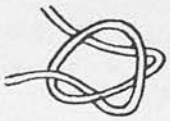
M.H.Sp. 1.

SECTION 1

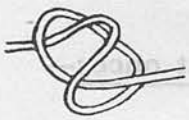
Do ALL the Questions on this page. Time allowed—5 minutes

Look at the drawings of a piece of string shown below. You have to say whether a knot would be left in the string if the ends were pulled tight. Answer questions 1 to 10 by writing YES or NO in the brackets under each question.

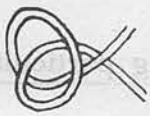
1. (.....)



2. (.....)



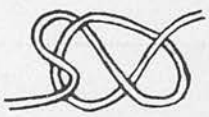
3. (.....)



4. (.....)

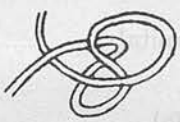
5. (.....)

6. (.....)

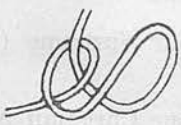


7. (.....)

8. (.....)



9. (.....)



10. (.....)

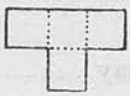
You next see drawings which are made up of small squares like the first drawing marked A. You have to say how many small squares, each the same size as A, would be needed to make up each drawing.

Look at the example. It is made up from four small squares like A, so 4 is the answer for the example.

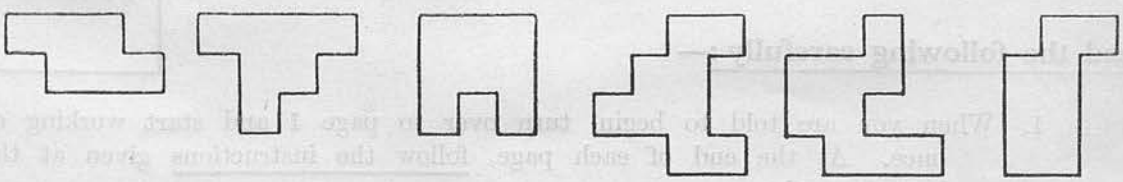
Now answer questions 11 to 22 by writing a number in the brackets under each drawing.

A

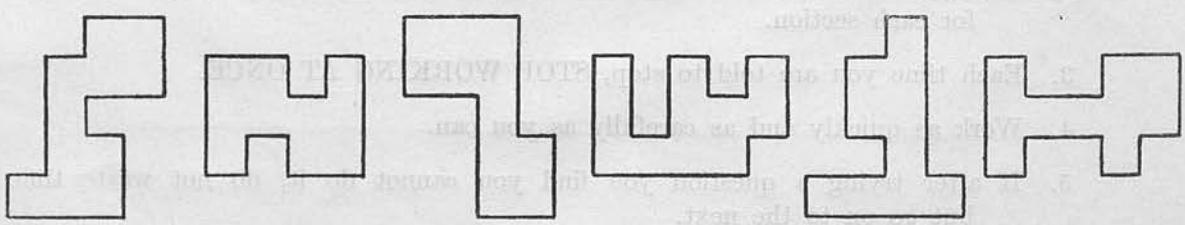
Example



(....4....)



11. (.....) 12. (.....) 13. (.....) 14. (.....) 15. (.....) 16. (.....)



17. (.....) 18. (.....) 19. (.....) 20. (.....) 21. (.....) 22. (.....)

DO NOT GO ON TO THE NEXT SECTION UNTIL YOU ARE TOLD

LOOK OVER YOUR WORK IN THIS SECTION TILL TIME IS UP

2

DO NOT TURN OVER UNTIL YOU ARE TOLD

SECTION 2

Do ALL the Questions on this page. Time allowed—7 minutes

Underneath the seven large letters which you see below there are small drawings. Each of these small drawings is part of one of the large letters. You have to find out which letter each drawing is taken from, and write this letter for your answer in the brackets under each drawing.

Look at the example. The drawing is part of the letter A, so A is the answer here.

Some of the letters may be used for more than one answer. You do not need any other letters than those shown, and the letters are to be used in the position shown.

Now answer questions 23 to 32.

Example  Answer (....A....)


23. (.....)


24. (.....)


25. (.....)


26. (.....)

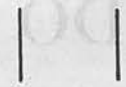

27. (.....)


28. (.....)

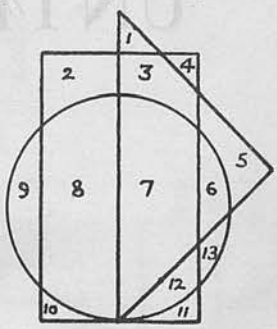

29. (.....)


30. (.....)


31. (.....)


32. (.....)

You next see a drawing of a circle, a rectangle or oblong, and a triangle. These three shapes are on top of one another so that 13 different spaces are formed, which are numbered 1 to 13. Some of these numbers are inside all three shapes—inside the circle, inside the rectangle and inside the triangle ; other numbers are inside two shapes ; and others are inside one shape only. For example, 9 is inside the circle only and 12 is inside both the circle and the rectangle, but not the triangle.



Now write your answers to these questions in the brackets.

33. Which are the two numbers inside the circle, but outside the rectangle and triangle ? . . . . . (..... : .....)
34. Which are the two numbers inside the triangle, but outside the circle and rectangle ? . . . . . (..... : .....)

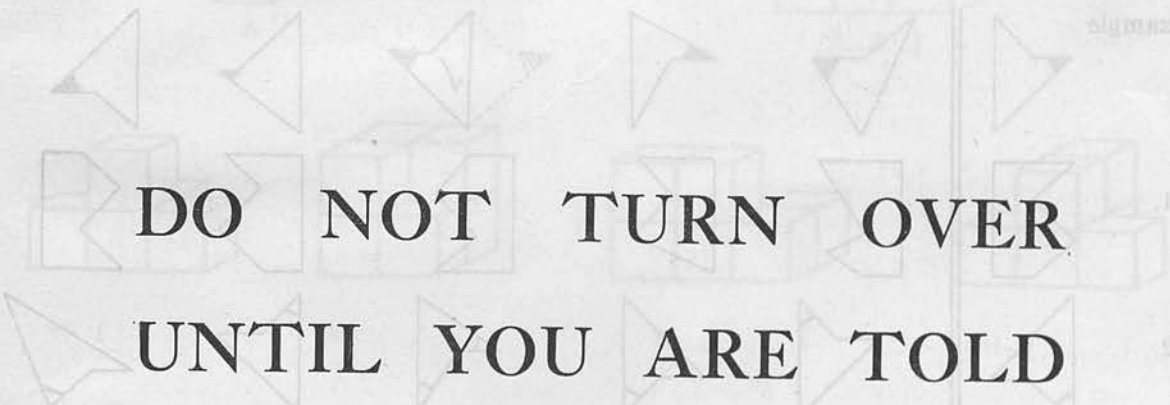
35. Which number is inside the circle, the rectangle and the triangle ? . (.....)
36. Which number is inside the triangle and rectangle, but outside the circle ? . . . . . (.....)

37. How many numbers are there inside the circle ? . . . . (.....)

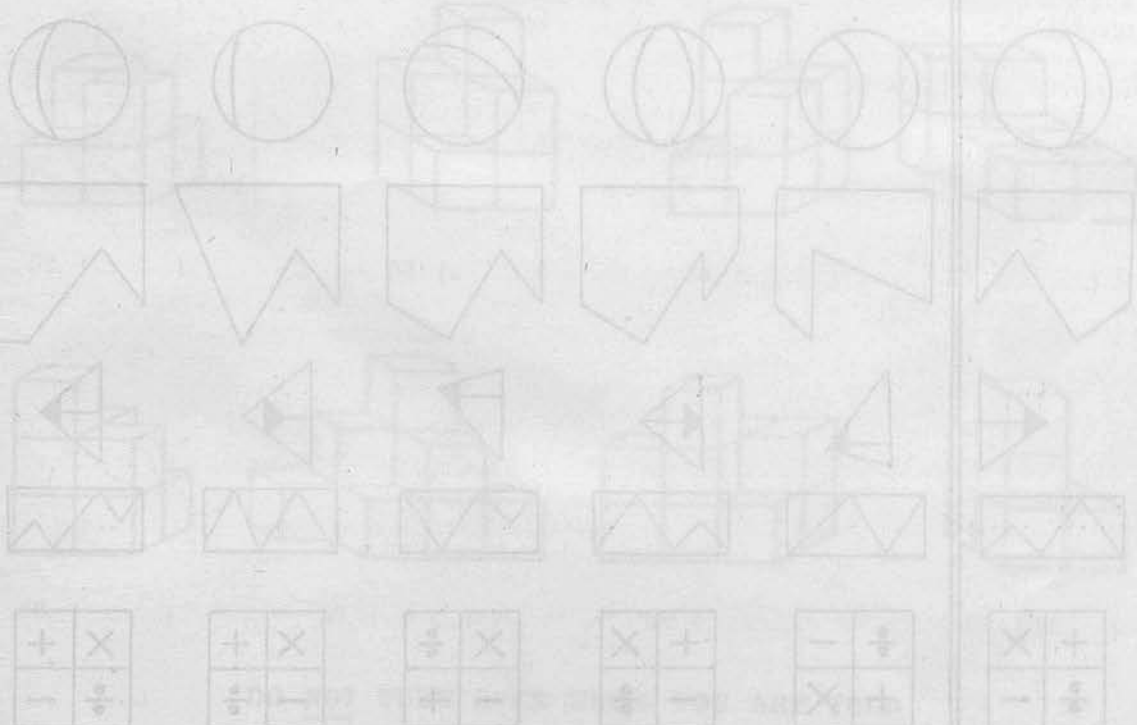
38. How many numbers are there outside the rectangle ? . . . (.....)
39. How many numbers are there inside the rectangle, but outside the circle and triangle ? . . . . . (.....)
40. How many numbers are there in one of the shapes only—that is, in not more than one shape ? . . . . . (.....)

DO NOT GO ON TO THE NEXT SECTION UNTIL YOU ARE TOLD

LOOK OVER YOUR WORK IN THIS SECTION TILL TIME IS UP



DO NOT TURN OVER UNTIL YOU ARE TOLD



SECTION 3

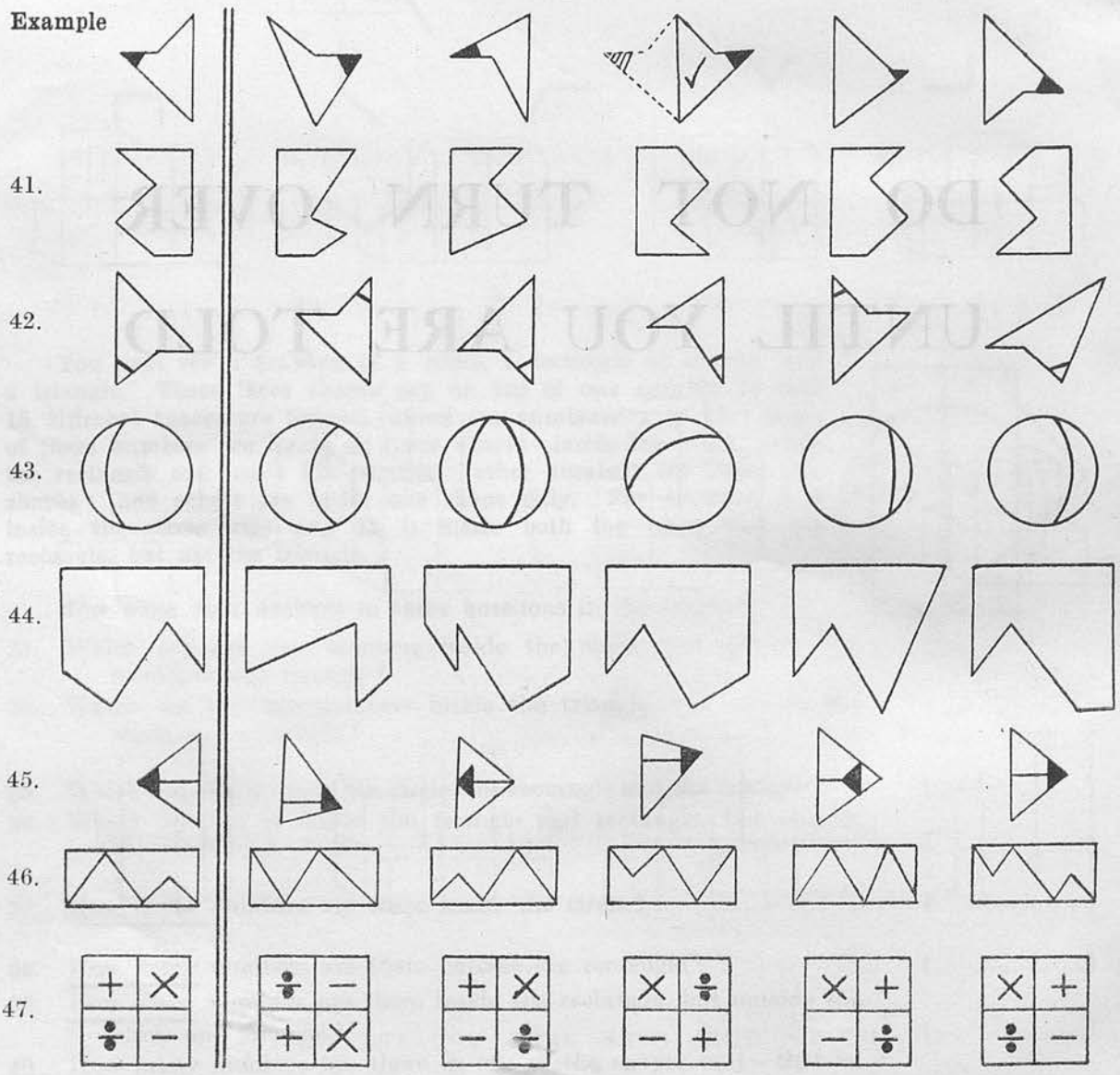
Do ALL the Questions on BOTH pages 5 and 6

Time allowed—6 minutes

For each drawing below on the left of the double line there are five drawings to the right. You have to put a tick (✓) inside one of the drawings on the right—the one which will be exactly the same as the drawing on the left when it is turned over like the page of a book.

Look at the example. The third drawing on the right of the double line would look the same as the example, if it were turned over. None of the other drawings would look like the example. So the third drawing is the answer and a tick is put inside it in this case.

Now do questions 41 to 47 by putting a tick (✓) INSIDE one drawing in each case.



GO ON TO PAGE 6 WITHOUT WAITING TO BE TOLD

SECTION 3—*continued*

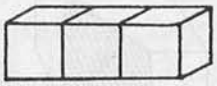
The drawings below are built up of small cubes like the first drawing marked **A**. You have to say how many cubes there are in each drawing. Remember that it may not be possible to see all the cubes at the back of some of the drawings, so that you may have to imagine them there. Count the number of cubes you can see and also those needed to support the upper cubes.

Look at the example. There are three small cubes in the drawing, so 3 is the answer for the example.

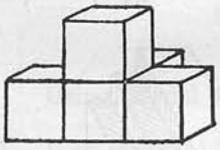
Now answer questions 48 to 59 by writing a number in the brackets under each drawing.

A

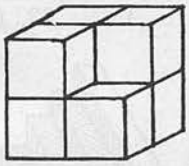
Example



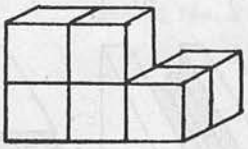
(....**3**....)



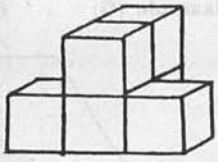
48. (.....)



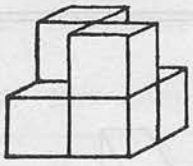
49. (.....)



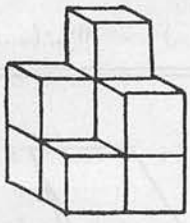
50. (.....)



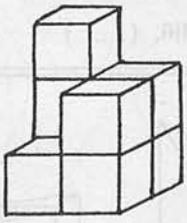
51. (.....)



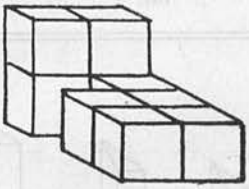
52. (.....)



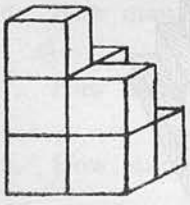
53. (.....)



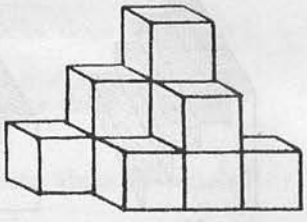
54. (.....)



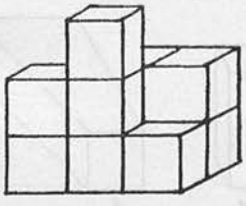
55. (.....)



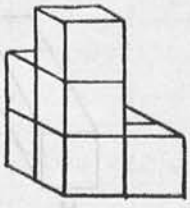
56. (.....)



57. (.....)



58. (.....)



59. (.....)

DO NOT TURN OVER UNTIL YOU ARE TOLD

LOOK OVER YOUR WORK IN THIS SECTION TILL TIME IS UP

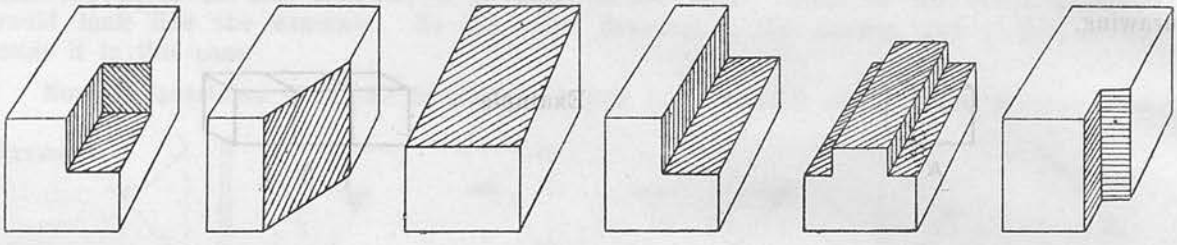
SECTION 4

Do ALL the Questions on BOTH pages 7 and 8. Time allowed—6 minutes

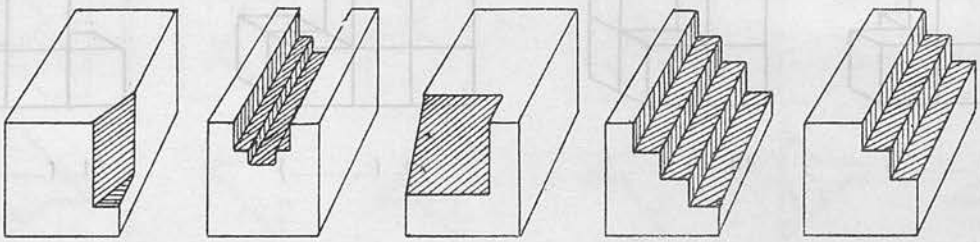
The drawings below are oblong blocks with a part cut away, leaving the surfaces which are shaded. Below the double line you see pieces marked with a letter and you have to find which piece will fit exactly the missing part of each block.

In the example shown, the piece G will fit the shaded part exactly, so G is the answer for the example.

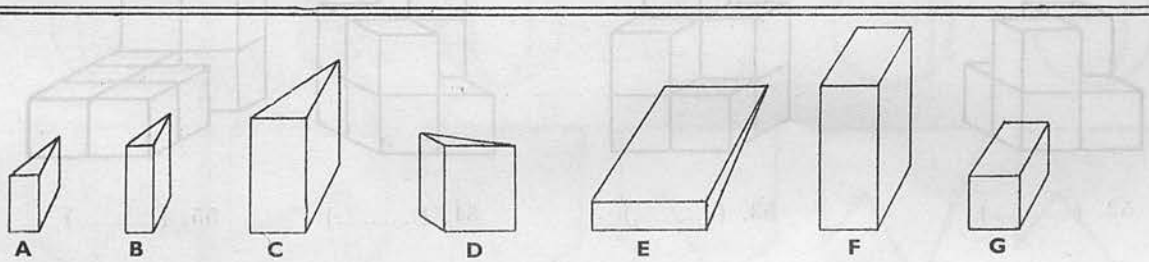
Now do questions 60 to 69. Remember that some of the pieces may have to be turned over before they will fit. Write one letter in the brackets under each question.

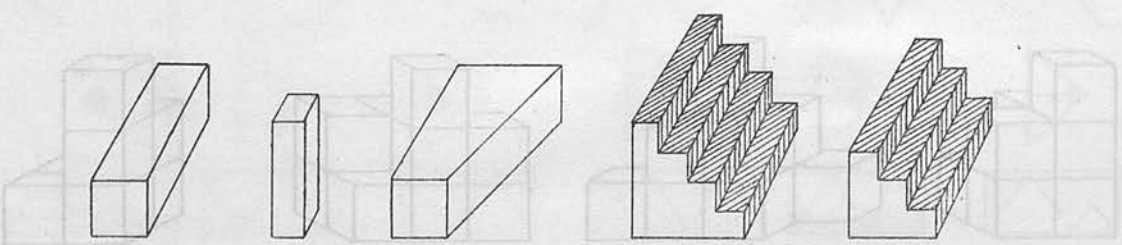


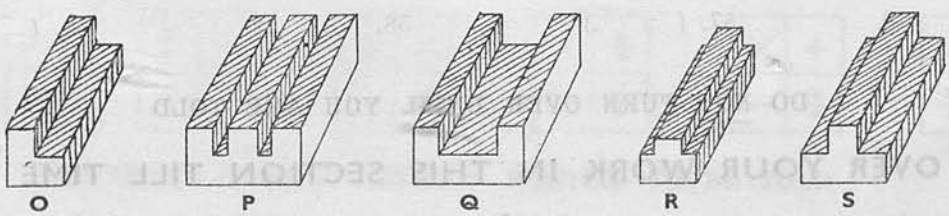
Example (G) 60. (.....) 61. (.....) 62. (.....) 63. (.....) 64. (.....)



65. (.....) 66. (.....) 67. (.....) 68. (.....) 69. (.....)







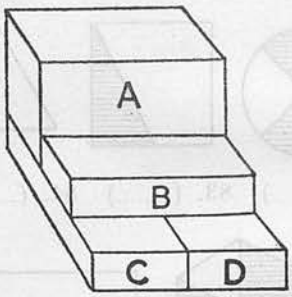
GO ON TO PAGE 8 WITHOUT WAITING TO BE TOLD

SECTION 4—*continued*

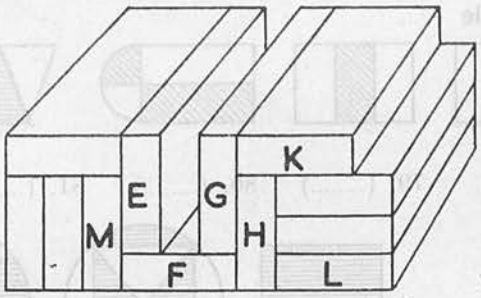
Below you see two models which have been built up from some blocks. You have to find out how many blocks are touched by the block mentioned in each question below.

For example, in Model No. 1, block B touches blocks A, C and D. B touches three blocks, so 3 is the answer here.

Now answer questions 70 to 78 by writing a number in the brackets after each question.



Model No. 1



Model No. 2

Using Model No. 1.

70. How many blocks does A touch ? . . . . . (.....)

71. How many blocks does D touch ? . . . . . (.....)

Using Model No. 2.

72. How many blocks does E touch ? . . . . . (.....)

73. How many blocks does F touch ? . . . . . (.....)

74. How many blocks does G touch ? . . . . . (.....)

75. How many blocks does H touch ? . . . . . (.....)

76. How many blocks does K touch ? . . . . . (.....)

77. How many blocks does L touch ? . . . . . (.....)

78. How many blocks does M touch ? . . . . . (.....)

DO NOT TURN OVER UNTIL YOU ARE TOLD

LOOK OVER YOUR WORK IN THIS SECTION TILL TIME IS UP

This test is based on the Block Counting Test of L. L. Thurstone ("Primary Mental Abilities," p. 31)

SECTION 5

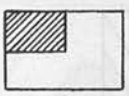
Do ALL the Questions on this page. Time allowed — 7 minutes

Each of the drawings below has a shaded part. You have to say what fraction of the whole drawing is shaded.

Look at the example. The shaded part is one-quarter of the whole drawing, so $\frac{1}{4}$ is the answer here.

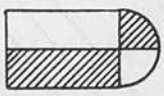
Now answer questions 79 to 90 by writing a fraction in the brackets under each drawing.

Example

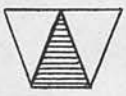


(....$\frac{1}{4}$....)

79. (.....)

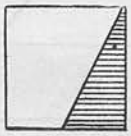


80. (.....)

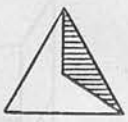


81. (.....)

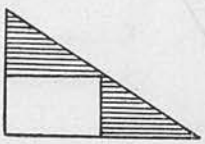
82. (.....)



83. (.....)

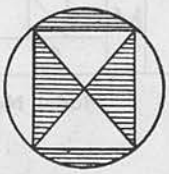


84. (.....)

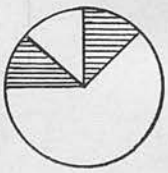


85. (.....)

86. (.....)

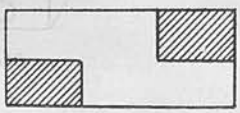


87. (.....)



88. (.....)

89. (.....)



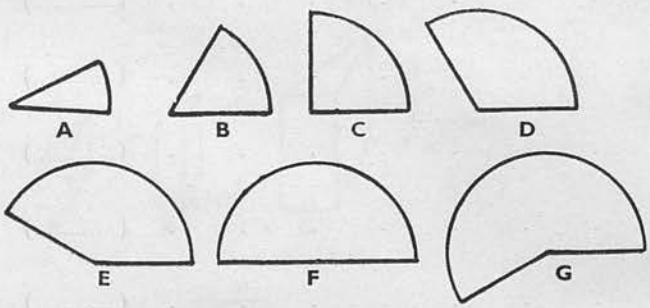
90. (.....)

You next see seven drawings marked A to G. You have either to add two of these drawings together or to subtract one from another, as you are told. The answer is one of the seven drawings shown, and its letter must be written for the answer.

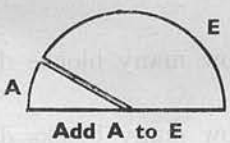
Look at the example below. Add A to E . . . (....F....)

F is the answer because when A is laid alongside E, the drawing F is made.

Now answer questions 91 to 100 by writing ONE letter in the brackets after each question.



Example



91. Add A to C . . . (.....)
92. Add A to B . . . (.....)
93. Add B to E . . . (.....)
94. Add C to D . . . (.....)
95. Add B to C . . . (.....)
96. Subtract C from D . . (.....)
97. Subrract A from E . . (.....)
98. Subtract B from F . . (.....)
99. Subtract C from G . . (.....)
100. Subtract D from F . . (.....)

LOOK OVER YOUR WORK IN THIS SECTION TILL TIME IS UP

Do NOT turn back to earlier pages

DO NOT TURN BACK
TO EARLIER PAGES